ON DIFFERENTIABLE MANIFOLDS WITH (ϕ, ψ)-STRUCTURES

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1. Introduction. In a previous paper,[1] we have defined for some odd dimensional manifolds two kinds of structures which we have called (ϕ, ξ, η)-structure and (ϕ, ξ, η, g)-structure. The latter is a (ϕ, ξ, η)-structure with a positive definite Riemannian metric g which stands in a notable relation with the (ϕ, ξ, η)-structure. These structures are remarkable in the sense that any differentiable manifold with (ϕ, ξ, η)-structure is an almost contact manifold and any almost contact manifold admits (ϕ, ξ, η, g)-structure.

In this paper, we shall study two kinds of structures for differentiable manifolds of any dimension, the first one ((ϕ, ψ)-structure) may be regarded as generalizations of almost complex structure, almost product structure and (ϕ, ξ, η)-structure, and the second one ((ϕ, ψ, g)-structure) may be regarded as generalizasions of almost Hermitian structure, almost product metric structure and (ϕ, ξ, η, g)-structure. We shall confine ourselves only to algebraic considerations, analytic considerations will be published in later papers.

2. (ϕ, ψ)-structures.

1°. Let M^n be a differentiable manifold of dimension n. Suppose first that there exist over M^n two tensor fields ϕ^i_j and ψ^i_j[2] of type (1, 1) which satisfy the following conditions:

$$\text{rank } |\phi^i_j| = l, \tag{2.1}$$

$$\text{rank } |\psi^i_j| = m, \tag{2.2}$$

$$\phi^i_j \psi^j_k = 0, \tag{2.3}$$

$$\psi^i_j \phi^j_k = 0, \tag{2.4}$$

1) S. Sasaki, On differentiable manifolds with certain structures which are closely related to almost contact structure I, Tôhoku Math. Journ. 12(1960) pp. 459-476.

2) We assume, unless otherwise stated, that the indices run the following range of integers:

$i, j, k, \alpha, \beta, \gamma = 1, 2, \ldots\ldots, n (= l+m)$,
$a, b, c = 1, 2, \ldots\ldots, l$,
$p, q, r = l+1, \ldots\ldots, n$
$A = 1, 2, \ldots\ldots, l'$, $\quad A^* = l'+A$,
$E, F = 1, \ldots\ldots, l_1$, $\quad H, K = l_1+1, \ldots\ldots, l (= l_1+l_2)$
$L = l+1, \ldots\ldots, l+m'$, $\quad L^* = l+m'+L$,
$M, N = l+1, \ldots\ldots, l+m_1$,
$S, T = l+m_1+1, \ldots\ldots, n$.