NOTE ON THE n-DIMENSIONAL TEMPERED ULTRA-DISTRIBUTIONS

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In this note, we shall describe explicitly the duality in the space of tempered ultra-distributions of J. Sebastião e Silva in the Euclidean n-space. And, as an application, we shall prove a theorem on the multiplication of tempered ultra-distributions.

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Notations: Let R^n (resp. C^n) be the real (resp. complex) n-space whose generic points are denoted by $x=(x_1,\dots\dots,x_n)$ (resp. $z=(z_1,\dots\dots,z_n)$). We shall use the notations: (i) $x+y=(x_1+y_1,\dots\dots,x_n+y_n)$, $\alpha x=(\alpha x_1,\dots\dots,\alpha x_n)$; (ii) $x\geqq 0$ means $x_1\geqq 0,\dots\dots,x_n\geqq 0$; (iii) $x\cdot y=\sum_{j=1}^n x_jy_j$ and (iv) $|x| = \sum_{j=1}^n |x_j|$.

Let p be a system of integers $\geqq 0$, $(p_1,\dots\dots,p_n)$. We shall denote by $|p|$ the sum $\sum_{j=1}^n p_j$ and by D^p the partial differential operator $\partial^{p_1+\dots+p_n}/\partial x_1^{p_1}\dots \dots\partial x_n^{p_n}$. We put, for any integer $k\geqq 0$, $\partial^k/\partial x^k=\partial^{nk}/\partial x_1^k\dots\dots\partial x_n^k$. $p+q$ is the system of integers $(p_1+q_1,\dots\dots,p_n+q_n)$. $p\geqq q$ means $p_1\geqq q_1,\dots\dots,p_n\geqq q_n$. Moreover, $x^p=x_1^{p_1}\dots\dots x_n^{p_n}$ and $x^k=x_1^k\dots\dots x_n^k$ (k an integer). For $p\geqq q$, put $\binom{p}{q}=\binom{p_1}{q_1}\dots\dots\binom{p_n}{q_n}$ with $\binom{p_j}{q_j}=p_j!/q_j!(p_j-q_j)!$.

We shall denote once for all by σ vectors $(\sigma_1,\dots\dots,\sigma_n)$ whose components are 0 or 1 and adopt the following conventions: (v) $(-1)^{|\sigma|}=(-1)^{\sigma_1+\dots+\sigma_n}$; (vi) $x^\sigma=((-1)^{\sigma_1}x_1,\dots\dots,(-1)^{\sigma_n}x_n)$ for any vector x; (vii) $k^\sigma=((-1)^{\sigma_1}k,\dots \dots,(-1)^{\sigma_n}k)$ for any integer k; (viii) $R^n_\sigma=\{x\in R^n: x^\sigma\geqq 0\}$; (ix) $C^n_{\sigma,\alpha} = \{z\in C^n: (-1)^{\sigma_1}\mathscr{I}z_1>\alpha,\dots\dots,(-1)^{\sigma_n}\mathscr{I}z_n>\alpha\}$ with $\alpha>0$ and (x) $\Delta_{\sigma,\alpha}$ is the path of integration $(-\infty+i(-1)^{\sigma_1}\alpha,\ \infty+i(-1)^{\sigma_1}\alpha)\times\dots\dots\times(-\infty+ i(-1)^{\sigma_n}\alpha,\ \infty+i(-1)^{\sigma_n}\alpha)$, oriented from $-\infty$ to $+\infty$. Finally V_α denotes the horizontal band in C^n defined by $V_\alpha=\{z\in C^n: |\mathscr{I}z_1|\leqq\alpha,\dots\dots,|\mathscr{I}z_n|\leqq\alpha\}$ with $\alpha>0$.

1. The basic spaces H and Λ_∞. Let H be the space of all C^∞-functions $\varphi(x)$ in R^n such that $\exp(k|x|)D^p\varphi(x)$ is bounded in R^n for any k and p. We define in H semi-norms