REMARK ON AUTOMORPHISMS IN CERTAIN COMPACT ALMOST HERMITIAN SPACES

CHEN-JUNG HSU

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1. Apte and Lichnerowicz [2] [5]*) have proved that if an affine transformation with respect to the first canonical connection $\overset{1}{\Gamma}_{ji}{}^{h}$ on a compact almost Hermitian manifold V_{2n} preserves the almost complex structure, then it is an automorphism, that is an isometry preserving the almost complex structure.

On the other hand, Tachibana [7] has recently proved that if a transformation on a compact O^*-space preserves the almost complex structure $\varphi_i{}^j$ and moreover the skew symmetric covariant tensor $\hat{K}_{kj} \equiv \overset{1}{K}{}_{kji}{}^{h}\varphi_h{}^i$, where $\overset{1}{K}_{kji}{}^{h}$ denotes the curvature tensor with respect to the first canonical connection $\overset{1}{\Gamma}_{ji}{}^{h}$, then it is an automorphism. By definition, an O^*-space considered by Kotō [3] is an almost Hermitian space on which the tensor $\nabla_k \varphi_{ji}$ is pure. Here ∇ denotes the covariant derivative with respect to the Christoffel symbol $\left\{ {h \atop ji} \right\}$ constructed from the Hermitian metric g_{ij} of V_{2n}. The tensor $\nabla_k \varphi_{ji}$ is called pure if it is pure with respect to any two indices, and $\nabla_k \varphi_{ji}$ is said to be pure with respect to indices k and j when the relation $(\nabla_r \varphi_{ji})\varphi_k{}^r = (\nabla_k \varphi_{ri})\varphi_j{}^r$ holds.

Some other special almost Hermitian spaces with additional conditions are considered by several authors. For example, an A-space considered by Apte [1] is by definition a space which fulfils the condition: $\nabla_r \varphi_i{}^r = 0$. K-space considered by Tachibana [6] is a space in which $\nabla_k \varphi_{ji} + \nabla_j \varphi_{ki} = 0$ holds. All these spaces are related each other in the following scheme [3]:

$$\begin{pmatrix}\text{almost}\\ \text{Hermitian}\\ \text{space}\end{pmatrix} \longrightarrow (A\text{-space}) \longrightarrow (O^*\text{-space}) \begin{matrix} \nearrow \begin{pmatrix}\text{almost}\\ \text{Kählerian}\\ \text{space}\end{pmatrix} \searrow \\ \searrow (K\text{-space}) \nearrow \end{matrix} \begin{pmatrix}\text{pseudo}\\ \text{Kählerian}\\ \text{space}\end{pmatrix}.$$

Now, let $\underset{v}{\pounds}$ denote the Lie derivative with respect to the infinitesimal transformation v^i, then $\underset{v}{\pounds}\varphi_h{}^i = 0$ if v^i preserves the almost complex structure.

*) Numbers in brackets refer to the reference at the end of the paper.