

REMARK ON AUTOMORPHISMS IN CERTAIN COMPACT ALMOST HERMITIAN SPACES

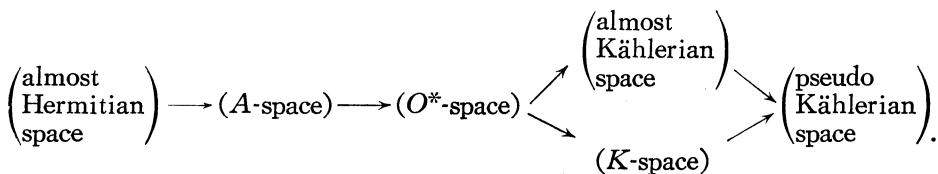
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1. Apte and Lichnerowicz [2] [5]^{*)} have proved that if an affine transformation with respect to the first canonical connection $\overset{1}{\Gamma}_{ji}{}^h$ on a compact almost Hermitian manifold V_{2n} preserves the almost complex structure, then it is an automorphism, that is an isometry preserving the almost complex structure.

On the other hand, Tachibana [7] has recently proved that if a transformation on a compact O^* -space preserves the almost complex structure φ_i^j and moreover the skew symmetric covariant tensor $\overset{1}{K}_{kj}{}^i \equiv \overset{1}{K}_{kji}{}^h \varphi_h^i$, where $\overset{1}{K}_{kji}{}^h$ denotes the curvature tensor with respect to the first canonical connection $\overset{1}{\Gamma}_{ji}{}^h$, then it is an automorphism. By definition, an O^* -space considered by Kotō [3] is an almost Hermitian space on which the tensor $\nabla_k \varphi_{ji}$ is pure. Here ∇ denotes the covariant derivative with respect to the Christoffel symbol $\left\{ \begin{smallmatrix} h \\ ji \end{smallmatrix} \right\}$ constructed from the Hermitian metric g_{ij} of V_{2n} . The tensor $\nabla_k \varphi_{ji}$ is called pure if it is pure with respect to any two indices, and $\nabla_k \varphi_{ji}$ is said to be pure with respect to indices k and j when the relation $(\nabla_r \varphi_{ji}) \varphi_k^r = (\nabla_k \varphi_{ri}) \varphi_j^r$ holds.

Some other special almost Hermitian spaces with additional conditions are considered by several authors. For example, an A -space considered by Apte [1] is by definition a space which fulfils the condition: $\nabla_r \varphi_i^r = 0$. K -space considered by Tachibana [6] is a space in which $\nabla_k \varphi_{ji} + \nabla_j \varphi_{ki} = 0$ holds. All these spaces are related each other in the following scheme [3]:



Now, let $\overset{v}{\mathcal{L}}$ denote the Lie derivative with respect to the infinitesimal transformation v^i , then $\overset{v}{\mathcal{L}} \varphi_h^i = 0$ if v^i preserves the almost complex structure.

^{*)} Numbers in brackets refer to the reference at the end of the paper.