APPLICATIONS OF FIBRE BUNDLES TO THE CERTAIN CLASS OF C^*-ALGEBRAS

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Introduction. It has been long discussed whether the sets of pure states of C^*-algebras are compact or not and some negative examples are found in the literature (cf. [5], [6], [13], [18]). However the question that which C^*-algebras have this property is remained unknown and it is the first motivation of our present paper to remove this obscurity. The result is the following one: *Let A be a C^*-algebra. If the set of pure states of A is compact and that of primitive ideals which are the kernels of one-dimensional irreducible representations forms an open set in the structure space of A, then A is isomorphic to the C^*-sum of a finite number of homogemeous C^*-algebras.*

A C^*-algebra is called *n-dimensionally homogeneous* if each irreducible representation of the algebra is n-dimensional. Such C^*-algebras were partly studied (without assuming a unit) in Kaplansky [10], [11] and Fell [4]. However, only a few results are known about the structure of these algebras. On the other hand, these algebras play an essential rôle in the construction of the composition series of GCR algebras. Thus the main part of the present paper is devoted to develop the structure theory of homogeneous C^*-algebras. Our method is somewhat different from the one usually employed in the literature. We use the theory of fibre bundles and illustrate the structure of homogeneous C^*-algebras in terms of fibre bundles.

Let A be an n-dimensionally homogeneous C^*-algebra and denote by $\Omega(A)$ the structure space of A. Let M_n and G be the $n \times n$ full matrix algebra and the group of all $*$-automorphisms of M_n. Then A defines a fibre bundle $\mathfrak{B}(A)$, called the structure bundle of A, over $\Omega(A)$ with fibre M_n and group G and A is represented as the C^*-algebra constructed by all cross-sections in $\mathfrak{B}(A)$. It is shown that the $*$-isomorphic relation between two n-dimensionally homogeneous C^*-algebras are equivalent to the equivalence relation between their structure bundles. Moreover, using the theory of bundles we can show that two algebraically isomorphic homogeneous C^*-algebras are necessarily $*$-isomorphic. Next we shall prove that the bundle $\mathfrak{B}$ over an arbitrary compact Hausdorff space with