ON THE MATSUSHIMA'S THEOREM IN A COMPACT EINSTEIN K-SPACE

SUMIO SAWAKI

(Received March 30, 1961 Revised July 21, 1961)

A. Lichnerowicz[1] has proved that the Matsushima's theorem[2] in a compact Kähler-Einstein space holds good in a compact Kählerian space with constant curvature scalar. In the previous paper [4], we have shown that the Matsushima's theorem is valid also in a compact almost-Kähler-Einstein space. The purpose of this paper is to show that it holds equally well in a compact Einstein K-space.

In §1 we shall give definitions and propositions. In §2 we shall give well known identities in a K-space. In §3 we shall prepare some lemmas on contravariant almost-analytic vectors in a K-space. The last §4 will be devoted to the proof of the main theorem.

1. Preliminaries. We consider a $2n$-dimensional almost-Hermitian space X_{2n} which admits an almost complex structure $\varphi_j^{\ i}$[3] and positive definite Riemannian metric tensor g_{ji} satisfying

$$\varphi_r^{\ i}\varphi_j^{\ r} = -\delta_j^i, \tag{1.1}$$

$$g_{rs}\varphi_j^{\ r}\varphi_i^{\ s} = g_{ji}. \tag{1.2}$$

By (1. 1) and (1. 2), we have

$$\varphi_{ji} = -\varphi_{ij}, \quad \nabla_h\varphi_{ji} = -\nabla_h\varphi_{ij} \tag{1.3}$$

where $\varphi_{ji} = \varphi_j^{\ r}g_{ri}$ and ∇_j denotes the operator of Riemannian covariant derivative.

We define the following linear operators

$$O_{ih}^{ml} = \frac{1}{2}(\delta_i^m\delta_h^l - \varphi_i^{\ m}\varphi_h^{\ l}), \qquad {}^*O_{ih}^{ml} = \frac{1}{2}(\delta_i^m\delta_h^l + \varphi_i^{\ m}\varphi_h^{\ l})$$

and a tensor is called pure (hybrid) in two indices if it is annihilated by

1) A. Lichnerowicz [1]. The number in brackets refers to Bibliography at the end of this paper.
2) Y. Matsushima [2].
3) As to the notations we follow S. Sawaki [3]. Indices run over $1, 2, \ldots, 2n$.