LINEAR CONNECTIONS AND QUASI-CONNECTIONS ON A DIFFERENTIABLE MANIFOLD

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1. Introduction. In the modern theory of linear connections on an n-dimensional differentiable manifold M, an important role is played by the frame bundle B over M, and by the n^2+n fundamental and basic vector fields E_λ^μ, E_α $(1 \leqq \alpha, \lambda, \mu \leqq n)$ on B. While the fundamental vector fields E_λ^μ are determined by the differential structure of M alone, the basic vector fields E_α together with the differential structure determine, and are determined by, a linear connection on M. The vector fields E_λ^μ and E_α are linearly independent everywhere on B and satisfy the following structure equations:

$$(1.1) \qquad \begin{aligned} [E_\lambda^\mu, E_\rho^\sigma] &= \delta_\rho^\mu E_\lambda^\sigma - \delta_\lambda^\sigma E_\rho^\mu, \\ [E_\alpha, E_\lambda^\mu] &= -\delta_\alpha^\mu E_\lambda, \\ [E_\alpha, E_\beta] &= -T_{\alpha\beta}^\gamma E_\gamma - R_{\mu\alpha\beta}^\lambda E_\lambda^\mu, \end{aligned}$$

where $[\ ,\ ]$ denotes the Lie product (bracket operation), δ_ρ^μ is the Kronecker delta, and $T_{\alpha\beta}^\gamma$, $R_{\mu\alpha\beta}^\lambda$ are functions on B corresponding to the torsion tensor and the curvature tensor on M of the linear connection.

Now equation $(1.1)_1$ merely expresses the Lie product in the Lie algebra of $GL(n, R)$, and equation $(1.1)_3$ determines the torsion and curvature of the linear connection. Therefore, among the equations (1, 1), only $(1.1)_2$ imposes any condition on the basic vector fields E_α. There arises then the natural question: The fundamental vector fields E_λ^μ being known, will any set of n vector fields E_α on B satisfying the condition

$$(1.1)_2 \qquad [E_\alpha, E_\lambda^\mu] = -\delta_\alpha^\mu E_\lambda$$

determine a linear connection on M?

In an attempt to answer this question, we discover a new kind of connections on M, to be called *quasi-connections*, which include the linear connections as particular case. More precisely, we shall obtain in this paper the following results:

i) With any set of n vector fields E_α on B satisfying $(1.1)_2$ there is associated