

ON SOME REPRESENTATIONS OF C^* -ALGEBRAS

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Introduction. Let $\pi = \int_{\Gamma} \pi_{\gamma} d\mu(\gamma)$ be an irreducible decomposition of a representation π of a C^* -algebra M over a measure space (Γ, μ) . Since traditionally we identify the representations within unitary equivalence, it comes into consideration whether the above decomposition can be regarded as a decomposition of the unitary equivalence class of π into the unitary equivalence class of $\pi(\gamma)$. Besides it is desirable from the view point of the duality that the decomposition can be constructed upon the structures which are completely determined by only M .

So G.W.Mackey, in [14], introduced the concept of the Borel structure in the dual space of the separable C^* -algebra and that of the separable locally compact group, in order to describe the behavior of the representations, especially the decompositions, on the dual space and this trying succeeded for the so-called C^* -algebras of type I with some well behaved (i.e., smooth) dual spaces. However we can not avoid some measure theoretic pathology for the C^* -algebras having rather badly behaved dual spaces.

Recently, the dual space of a C^* -algebra has been studied successively by several authors: J.Dixmier [2, 3], J.M.G. Fell [6, 7, 8], J.Glimm [9], M.A. Guichardet [10, 11], J.Tomiyama [26] and J.Tomiyama-M. Takesaki [25]. Among them, J.Glimm [9] obtained the extremely fruitful results for the relation between the dual space and the structure of a separable C^* -algebra, which says that a separable C^* -algebra has the well behaved (smooth) dual space if and only if it has only representations of type I, and that a separable C^* -algebra is of type I if and only if it is GCR-algebra.

It seemed to be the second step to see what happens for the representations of C^* -algebras with *badly behaved* dual spaces. M.A.Guichardet [10] showed that the representations of type I behave well in their irreducible direct integral decompositions. Moreover, in [11], he gave an example of the C^* -algebra having disjoint factor representations of type II_1 with same kernels.

Thus the present paper is devoted to show that *for a separable C^* -algebra with badly behaved dual space there exists a continuum family of disjoint factor representations of type II and of type III respectively with the same kernel and moreover there are representations θ^i of type i , $i = II, III$, with the following properties: there exist irreducible direct integral decomposition of θ^i ,*