SOME PROPERTIES OF MANIFOLDS WITH CONTACT METRIC STRUCTURE

YOJI HATAKEYAMA, YOSUKE OGAWA AND SHUKICHI TANNO

(Received December 11, 1962)

Introduction. Recently S.Sasaki [2][1] defined the notion of (ϕ, ξ, η, g)-structure of a differentiable manifold and showed that the structure is closely related to almost contact structure defined by J.W.Gray [1]. Further he and one of the authors [3] defined four tensors $N^i{}_{jk}$, $N^i{}_j$, N_{jk} and N_j associated with this structure and enumerated relations connecting these tensors. Especially N_{jk} and N_j vanish identically when the structure is the one associated to contact structure, or so-called contact metric structure. And it was shown that the vanishment of $N^i{}_{jk}$ implies the vanishment of all other tensors $N^i{}_j$, N_{jk} and N_j, and that in the case of contact metric structure the vanishment of $N^i{}_j$ is equivalent to the fact that the vector field ξ^i is a Killing vector field.

In this note we call contact metric structure with vanishing $N^i{}_j$ or $N^i{}_{jk}$ K-contact metric structure or normal contact metric structure respectively, and we shall study some conditions for a manifold with almost contact metric structure or a Riemannian manifold to admit such structure.

1. Conditions for manifolds to admit K-contact metric structure. In this section, we shall study the case of K-contact metric structure, i. e., the case such that the associated vector field ξ^i is a Killing vector field. We shall begin with the following

LEMMA 1. *Suppose ξ^i be a Killing vector field on an m-dimensional Riemannian manifold M^m, then the relations*

$$\xi^i{}_{,j,k} = R^i{}_{jhk}\xi^h \tag{1. 1}$$

hold good, where commas mean the covariant differentiation with respect to the Riemannian connection and $R^i{}_{jhk}$ is the curvature tensor.

PROOF. Since ξ^i is a Killing vector field, we have

$$\pounds(\xi)g_{ij} = 0,$$

where $\pounds(\xi)$ means the Lie derivation with respect to the infinitesimal transformation ξ^i, which implies

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1) Numbers in brackets refer to the bibliography at the end of the paper.