SOME NOTES ON DIFFERENTIABLE MANIFOLDS WITH ALMOST CONTACT STRUCTURES

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1. Introduction. By an almost contact structure, we mean the (ϕ, ξ, η)-structure defined by Prof. S.Sasaki [4][1], i.e., the structure defined by tensor fields ϕ^i_j, ξ^i and η_j satisfying the following relations,

(1. 1) $$\xi^i\eta_i = 1,$$

(1. 2) $$\phi^i_j\xi^j = 0,$$

(1. 3) $$\phi^i_j\phi^j_k = -\delta^i_k + \xi^i\eta_k.$$

In this paper, we first consider a principal fibre bundle over a manifold with an almost complex structure with a group A^1, where A^1 denotes a 1-dimensional Lie group. And making use of the almost complex structure and an infinitesimal connection in this principal fiber bundle, we shall define an almost contact structure on the bundle space such that the 1-form defined by η_j coincides with the connection form. Next, we shall study the condition for the normality of this almost contact structure, where a normal almost contact structure means the structure such that the tensor defined in [5] vanishes identically. In particular, in the case of a regular contact structure on a compact manifold, we get the result that a necessary and sufficient condition for the manifold to admit a normal contact metric structure associated with the contact structure is that the base manifold of Boothby-Wang's fibering (Cf. [1], Theorems 2 and 3) is a Hodge manifold.

2. Definition of almost contact structures in bundle spaces. Let M' be a $2n$-dimensional differentiable manifold with an almost complex structure defined by a tensor field F of type (1, 1) satisfying the relation

(2. 1) $$F^2_xX = -X \qquad \text{for } x \in M',\ X \in T_x(M'),$$

where $T_x(M')$ denotes the tangent vector space of M' at x.

We consider a principal fiber bundle over M' with group A^1, and denote its bundle space and projection by M and p respectively. Now, we suppose that an infinitesimal connection in $M(M', p, A^1)$ is given and we denote its

1) Numbers in square brackets refer to the bibliography at the end of the paper.