ON THE UNITARY EQUIVALENCE AMONG THE COMPONENTS OF DECOMPOSITIONS OF REPRESENTATIONS OF INVOLUTIVE BANACH ALGEBRAS AND THE ASSOCIATED DIAGONAL ALGEBRAS

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Introduction. Let $\varphi = \int_{\Gamma}^{\oplus} \varphi(\gamma) d\mu(\gamma)$ be an irreducible decomposition of a representation φ of an involutive Banach algebra $\mathfrak{B}$ over a measure space (Γ, μ). As shown by several authors in [4], [8], [9], [13] etc., this decomposition cannot be regarded as a decomposition of the unitary equivalence class of φ into the unitary equivalence classes of $\varphi(\gamma)$ except for some fairly nice cases, whereas this decompositon is determined only up to unitary equivalence. For instance, some representations can be decomposed in two ways that have no common components as in [8] and some two representations of quite different types can be decomposed into the direct integrals of the same components as in [13]. Therefore it comes into considerations what determines the unitary equivalence relation among the components $\{\varphi(\gamma) : \gamma \in \Gamma\}$ of the decomposition $\varphi = \int_{\Gamma}^{\oplus} \varphi(\gamma) d\mu(\gamma)$. For this question we shall answer in § 1 that the algebraic relation between the commutant $\varphi(\mathfrak{R})' = \boldsymbol{M}$ of $\varphi(\mathfrak{B})$ and the associated diagonal algebra $\boldsymbol{A}$ determines completely the unitary equivalence relation $\mathfrak{R}$ among the components $\{\varphi(\gamma) : \gamma \in \Gamma\}$. So we can regard $\mathfrak{R}$ as an algebraic invariant of the couple $(\boldsymbol{M}, \boldsymbol{A})$. A. Guichardet used $\mathfrak{R}$ for characterization of discrete von Neumann algebras in [5]. We study the behavior of $\mathfrak{R}$ in more general situation. In § 2 we shall give the definitions of simplicity, smoothness and complete roughness of $\boldsymbol{A}$ in $\boldsymbol{M}$ using $\mathfrak{R}$. In § 3 we shall reduce the study of smooth maximal abelian subalgebras to that of simple ones. § 4 is devoted to show some relations between simple or completely rough maximal abelian subalgebras and regular, semi-regular or singular ones defined in [3]. Finally in § 5 we shall give some examples of factors of type II and type III with simple maximal abelian subalgebras and completely rough ones simultaneously respectively.

1. Unitary equivalence relation. Let Γ be a standard Borel space[1)] and μ a Borel measure on Γ. Let $\boldsymbol{A} = L^{\infty}(\Gamma, \mu)$ be the commutative von Neumann

1) If a Borel space $(\Gamma, \mathscr{B})$ is Borel isomorphic to some separable complete metric space equipped with the Borel structure generated by closed sets, then we call it standard according to Mackey[9]. Calling the member of $\mathscr{B}$ Borel set, we shall omit the letter $\mathscr{B}$.