AN EXTENSION OF THE INTERPOLATION THEOREM OF MARCINKIEWICZ II

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1. Introduction. The purpose of this paper is to give the detailed proof of the theorem which is announced in my previous paper [2], that is, to show that Marcinkiewicz's theorem on the interpolation of operators (e.g.see A.Zygmund [6; Chap. XII]) holds good for Hardy class $H_p(p \geqq 1)$.

H_p-class (p > 0) is the space of all functions analytic in the unit circle such that

$$\|\varphi\|_p = \lim_{r\to 1-0} \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} |\varphi(re^{i\theta})|^p d\theta \right\}^{1/p}$$

is finite. For $p \geqq 1$ this class is equivalent to the space of functions in $L_p(-\pi, \pi)$ with the ordinary L_p-norm such that their Fourier expansion is power series type, that is,

$$\sum_{n=0}^{\infty} a_n e^{inx}. \tag{1.1}$$

Our method of proof depends on the real one and can be applied to some n-dimensional analogues of H_p-class.

Sections 2 and 3 contain the case of one variable H_p-space.

Sections 4 and 5 treat n-dimensional analogues of H_p-space.

Section 6 contains some applications to the theorems on Fourier series.

2. Two Lemmas. We begin by defining some notations. Let $f \in L_p(-\pi, \pi)(p > 1)$ be periodic with period 2π and its Fourier expansion be

$$f(x) \sim \sum_{n=-\infty}^{\infty} a_n e^{inx},$$

then its conjugate function $\tilde{f}(x)$ is defined by

$$\tilde{f}(x) = \lim_{\varepsilon\to 0} \frac{1}{\pi} \int_{\pi \geqq |y| \geqq \varepsilon} \frac{f(y)}{2\tan(x-y)/2} dy \tag{2.1}$$

or equivalentely

$$\tilde{f}(x) \sim -\sum_{n=-\infty}^{\infty} i\,(\text{sign}\, n)\, a_n e^{inx}. \tag{2.2}$$

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