SOME TRANSFORMATIONS ON MANIFOLDS WITH ALMOST CONTACT AND CONTACT METRIC STRUCTURES, II

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For the notations we conform to the previous paper [15][1)] which is the part I of the present paper. By Φ we denote the group of all diffeomorphisms which leave ϕ of the structure tensors (ϕ, ξ, η) invariant in an almost contact manifold M. We assume always that the dimension of the manifold is greater or equal to 3. In §4, we shall see that Φ is a Lie transformation group if M is a contact Riemannian manifold, and some structures of this group are considered. In §6, for an arbitrary point x of a contact Riemannian manifold M and an element μ of Φ, we shall search for the relation between the scalar curvature R_x at x and $R_{\mu x}$ at μx by lengthy calculations. As applications, in §7, we treat some contact Riemannian manifolds which are supposed to satisfy certain conditions, for examples, being of constant scalar curvature, or being an Einstein space, etc.. Then, with some exceptions, it is shown that Φ coincides with the group of all automorphisms.

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4. The group Φ on contact Riemannian manifolds.

THEOREM 4-1. *For every contact Riemannian manifold, the group Φ is a Lie transformation group.*

PROOF. We denote by L_ϕ the Lie algebra of all infinitesimal transformations leaving ϕ invariant and by $\mathfrak{A}$ that of all infinitesimal automorphisms. Then, it has been shown [16] that L_ϕ is finite dimensional. In fact, $L_\phi = \mathfrak{A} + L$ (direct sum), where L is a 1-dimensional Lie algebra generated by a vector field $Z \in L_\phi$ such that $\mathfrak{L}(Z)\eta = \eta$. Moreover, we have

$$[\mathfrak{A},\mathfrak{A}] \subset \mathfrak{A},\ [\mathfrak{A},L] \subset \mathfrak{A}. \tag{4. 1}$$

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1) Correction of [15]; we must insert the underlined part into the conclusion of the statement of Corollary (p. 141) as follows:
μ is necessarily an isometry. Therefore, $\underline{\text{if } \mu\xi \neq -\xi,}$ μ is an automorphism of this almost contact metric structure.
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