STRONG SUMMABILITY OF WALSH FOURIER SERIES

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1. Introduction. Let $f(x)$ be an integrable and periodic function with period 1. Let $\{\psi_n(x)\}$ $(n=0, 1, 2, \cdots)$ be the orthogonal system of Walsh (We refer to [4] for definition of the system), and

$$(1) \qquad \sum_{n=0}^{\infty} a_n \psi_n(x), \qquad a_n = \int_0^1 f(x)\psi_n(x)\,dx.$$

be the Walsh Fourier series of $f(x)$. We denote the partial sum of (1) by

$$s_n(x) = \sum_{\nu=0}^{n-1} a_\nu \psi_\nu(x)$$

and the strong Cesàro mean of (1) by

$$R_n^\delta(x) = \frac{1}{A_n^\delta}\sum_{\nu=0}^{n} A_{n-\nu}^{\delta-1}|s_\nu(x)|,$$

where

$$A_n^\delta = \binom{n+\delta}{n} \cong \frac{n^\alpha}{\Gamma(\alpha+1)}.$$

Paley [4] stated the following theorem without proof.

THEOREM A. *If $f(x)$ belongs to the class $L^p(p>1)$ and $\delta>1/p$, then**)

$$\int_0^1 (\max_{0\leqq n<\infty} |R_n^\delta(x)|)^p\,dx \leqq A_p \int_0^1 |f(x)|^p\,dx.$$

And he conjectured that Theorem A would be valid for any $\delta>0$.

In the present note, the author will prove this conjecture in stronger form. In fact, if we set

*) A_p is a constant depending on p only and is not necessarily the same in different occurences.