A COMPLEMENT TO "ON THE UNITARY EQUIVALENCE AMONG THE COMPONENTS OF DECOMPOSITIONS OF REPRESENTATIONS OF INVOLUTIVE BANACH ALGEBRAS AND THE ASSOCIATED DIAGONAL ALGEBRAS"

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After the publication of my paper, indicated in the title, I obtained the following complementary results of Remark in p.370 and Theorem 3.4.

THEOREM. *Let $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ be two maximal abelian subalgebras of a von Neumann algebra $\boldsymbol{M}$. Let e_1 and e_2 be non-zero projections of $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ respectively. If there exists a central projection z of $\boldsymbol{M}$ such that $e_1 \leqq z$ and $e_2 \leqq (I - z)$, then $\boldsymbol{A}_1 e_1$ and $\boldsymbol{A}_2 e_2$ are unrelated. Hence if $\boldsymbol{A}_1 e_1$ and $\boldsymbol{A}_2 e_2$ are similar, then e_1 and e_2 have the same central carrier.*

PROOF. We choose a weakly dense uniformly separable C^*-subalgebra $\mathfrak{A}$ of $\boldsymbol{M}'$ which contains z. Let $\boldsymbol{A}_1 = L^\infty(\Gamma_1, \mu_1)$ and $\boldsymbol{A}_2 = L^\infty(\Gamma_2, \mu_2)$. Let E_1 and E_2 be the Borel subsets of Γ_1 and Γ_2 associated with e_1 and e_2 respectively. If we decompose the underlying Hilbert space $\mathfrak{H}$ of $\boldsymbol{M}$ and the operator x of $\mathfrak{A}$ with respect to $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ as follows;

$$\mathfrak{H} = \int_{\Gamma_1}^{\oplus} \mathfrak{H}^1(\gamma_1)\, d\mu_1(\gamma_1), \qquad \mathfrak{H} = \int_{\Gamma_2}^{\oplus} \mathfrak{H}^2(\gamma_2) d\mu_2(\gamma_2),$$

and

$$x = \int_{\Gamma_1}^{\oplus} x^1(\gamma_1) d\mu_1(\gamma_1), \qquad x = \int_{\Gamma_2}^{\oplus} x^2(\gamma_2)\, d\mu_2(\gamma_2).$$

Then we have $z^1(\gamma_1) = I$ for every $\gamma_1 \in E_1$ and $z^2(\gamma_2) = 0$ for every $\gamma_2 \in E_2$ by elimination of null sets from E_1 and E_2. Hence $\mathfrak{R}_{A_1, A_2}^{M, \mathfrak{A}, \Phi^1, \Phi^2}(\gamma_1, \gamma_2)$ does not hold for every $(\gamma_1, \gamma_2) \in E_1 \times E_2$, where Φ^i is the family of the representation of $\mathfrak{A}$ defined by $x \rightarrow x^i(\gamma_i)$ $(i = 1, 2)$. The second assertion is a direct consequence of the first.

As an interpretation of the above theorem, we get the following

COROLLARY. *Let φ_1 and φ_2 be two representations of an involutive Banach algebra $\mathfrak{B}$ over Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$. Decompose φ_1 and φ_2 into direct integrals of irreducible representations over some standard measure*