SUMMABILITY METHODS OF BOREL TYPE AND TAUBERIAN SERIES

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1. Introduction. Let $t_p = \sum_{k=0}^{n} c_{pk}s_k$ denote a linear transformation of a sequence $s_n = \sum_{k=0}^{n} u_k$ where $\{u_k\}$ is a real or complex sequence. When a sequence $\{u_n\}$ satisfies Tauberian condition of the form $\lambda_n u_n = O(1)$[1], it is sometimes possible to estimate $\limsup |t_p - s_n|$ even when $\{s_n\}$ and $\{t_p\}$ are divergent. Such estimation was initiated by H. Hadwiger [5]. R. P. Agnew [1], [2], [3] and [4] gave such estimations for Borel, Abel and integral transforms.

In a recent paper, A.Meir [7] defined summability methods of Borel type $B(a, q)$ which contained Borel, Valiron, Euler, Taylor and S_α transformation and showed the following fact:

If $t_p = \sum_{k=0}^{\infty} c_{pk}s_k$ belongs to $B(a, q)$,

$$\limsup_{\alpha\to\infty} |\sqrt{n}\, u_n| = L < +\infty \tag{1. 1}$$

and $n = n(\alpha)$, $p = p(\alpha)$ are positive increasing functions tending to $+\infty$ as $\alpha\to\infty$ such that

$$\limsup_{\alpha\to\infty} |n - q|/\sqrt{q} = M < +\infty, \tag{1. 2}$$

then

$$\limsup_{\alpha\to\infty} |t_p - s_n| \leqq A\cdot L, \tag{1. 3}$$

where A is a finite constant depending only on M.

In the present paper, the author will consider the case

$$\limsup_{\alpha\to\infty} |n - q|/\sqrt{q} = +\infty$$

1) We have $\lambda_n = \sqrt{n}$ for Borel transforms and $\lambda_n = n$ for Abel transforms.