ON PURELY-TRANSCENDENCY OF CERTAIN FIELDS

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(Received March 7, 1964)

Let $K = k(x_1, x_2, \cdots, x_n)$ be a purely transcendental field over k of transcendence degree n. Now we set $\sigma x_j = x_{j+1}$ for any integer j with $1 \leqq j \leqq n - 1$, while $\sigma x_n = x_1$. Then σ induces an automorphism of K over k, which will be also denoted by the same letter σ. If we denote by L the set of all elements of K which remain fixed under σ, then K is a normal extension of degree n with Galois group $\mathfrak{G} = \{I, \sigma, \sigma^2, \cdots, \sigma^{n-1}\}$. Now it arises the question "Is L purely transcendental over k?", which is refered to as "Chevalley's Problem" by Dr. K. Masuda. Let p be the characteristic of k and assume that p does not divide n. For any positive integer n less than or equal to 7, he proved that L is purely transcendental over k [1]. On the same Journal of Nagoya, Dr. Kuniyoshi proved the purely-transcendency for the case of nonzero characteristic p and $n = p$ [2]. Let r be any positive integer and $p(>0)$ the characteristic of k. Dr. Kuniyoshi proved further that L is purely transcendental over k, if $n = p^r$ [3].

All the fields treated in this article are assumed to be of characteristic zero.

Already E. Noether showed that L is purely transcendental over k, if the ground field k contains a primitive n-th root ζ. Accordingly one of our concern is to diminish this restriction concerning the ground field. Now we can state the following main theorem:

MAIN THEOREM. *L is purely transcendental over k, if one of the following conditions holds, where l is an odd prime number:*

(i) *$n = l^{2r}$ and k contains a primitive l^r-th root of unity,*

(ii) *$n = l^{2r+1}$ and k contains a primitive l^{r+1}-th root of unity,*

(iii) *$n = 2^{2r}$ and k contains a primitive 2^{r+1}-th root of unity,*

(iv) *$n = 2^{2r+1}$ and k contains a primitive 2^{r+1}-th root of unity.*

As a special case it holds that L is purely transcendental if $n = 3^3$ and k contains primitive 3^2-th roots of unity. But more precisely we can assert the same result under the weaker condition that the ground field contains merely cube roots of unity. Dr. Kuniyoshi conjectures that more generally following fact will hold:

"If $n = p^r$ (p : prime number) and k contains p-th roots of unity, then L is purely transcendental over k."