AN EXPLICIT REPRESENTATION OF THE GENERALIZED PRINCIPAL IDEAL THEOREM FOR THE RATIONAL GROUND FIELD

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In the following lines the author wants to give an explicit representation for generalized principal ideal theorems of S.Iyanaga [1] and T.Tannaka [2] for the case of rational ground field.

Let K be the "Strahlklassenkörper" over k, with "Geschlechtermodul" $\mathfrak{F} = \mathfrak{F}(K/k)$, then every ideal $\mathfrak{a}$ of k which is unramified in K, becomes principal ideal belonging to the principal class modulo $\mathfrak{F}$ (Iyanaga [1]).

Tannaka [2] obtained, suggested by a conjecture of Prof. Deuring, a more precise form of the principal ideal theorem, he gave namely those bases $\theta(\mathfrak{a})$ of $\mathfrak{a}$ (unramified ideals in k), for which the units

$$\varepsilon(\mathfrak{a}, \mathfrak{b}) = \frac{\theta(\mathfrak{a})\ \theta(\mathfrak{b})^{\sigma(\mathfrak{a})}}{\theta(\mathfrak{a}\mathfrak{b})}$$

lie in the ground field. There $\sigma(\mathfrak{a}) = (K/k, \mathfrak{a})$ means the Artin-automorphism of $\mathfrak{a}$.

Let now n,m be two natural numbers which are relatively prime to each other, $\zeta_n = \exp\left(\frac{2\pi i}{n}\right)$ and $\mathfrak{F}_n$ the "Geschlechtermodul" of $Q(\zeta_n)/Q$ (Q: rational number field), then we can find a unit $\boldsymbol{E}(m)$ in $Q(\zeta_n)$ explicitly, for which

$$m \equiv \boldsymbol{E}(m) \pmod{\mathfrak{F}_n}$$

and

$$\frac{\boldsymbol{E}(m)(\boldsymbol{E}(m'))^{\sigma(m)}}{\boldsymbol{E}(mm')} = 1$$

hold.

1. Calculation of the "Geschlechtermodul". Let $n = p_1^{e_1} p_2^{e_2} \cdots p_t^{e_t} = n_1 n_2 \cdots n_t$ be a natural number, where $p_1, p_2, \cdots, p_t$ are different prime numbers and $p_1 = 2$, $e_1 = 0$ or $e_1 = 2$, and $\mathfrak{F}_n$ the "Geschlechtermodul" of $Q(\zeta_n)/Q$. We have then