ON ALMOST CONTACT METRIC MANIFOLDS ADMITTING PARALLEL FIELDS OF NULL PLANES

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1. Introduction. Since Prof. S. Sasaki [3], [4][1] introduced the notion of (φ, ξ, η, g) structure in odd dimensional manifolds which is equivalent to the almost contact metric structure, many authors have investigated it vigorously from differential geometric point of view.

In the first place it is seen easily that the almost contact metric manifold admits a certain field of complex null planes (we call it π-plane field) and that the π-plane field is parallel if and only if the tensor $\varphi^i{}_j$ is covariant constant. The main purpose of this paper is to study on the geometry of the almost contact metric manifold M^{*2n+1} of dimension $2n+1$ whose π-plane field is parallel.

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2. An almost contact metric manifold admitting certain parallel fields of complex null n-planes. Let M^{2n+1} be an almost contact metric manifold, then there exist tensor fields $\varphi^i{}_j$, ξ^i, η_j[2] and positive definite Riemannian metric tensor g_{ij} over M^{2n+1} satisfying

$$(2.1)\quad \begin{cases} \text{rank } |\varphi^i{}_j| = 2n, \quad \varphi^i{}_j\xi^j = 0, \quad \varphi^i{}_j\eta_i = 0, \quad \xi^i\eta_i = 1, \\ \varphi^i{}_j\varphi^j{}_k = -\delta^i{}_k + \xi^i\eta_k, \quad g_{ij}\varphi^i{}_h\varphi^j{}_k = g_{hk} - \eta_h\eta_k, \quad \eta_i = g_{ij}\xi^j. \end{cases}$$

It is obvious that the tensor $\varphi^i{}_j$ satisfies $\varphi^i{}_j\varphi^j{}_k\varphi^k{}_h = -\varphi^i{}_h$, and hence the rank of the matrix $(\gamma^i{}_j) = (\varphi^i{}_j - \sqrt{-1}\ \delta^i{}_j)$ is $n+1$ over M^{2n+1}. Now we consider vectors λ^j satisfying

$$(2.2)\qquad \gamma^i{}_j\lambda^j = 0,$$

and take a field of n-planes π^n over M^{2n+1} spanned by the λ^j's, which is called π-plane field hereafter. From (2.2) we see directly for any vector $\lambda^i{}_{(\alpha)}$ in π-plane field

$$\eta_i\lambda^i{}_{(\alpha)} = 0, \qquad g_{ij}\lambda^i{}_{(\alpha)}\lambda^j{}_{(\beta)} = 0.$$

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1) Numbers in blackets refer to the references at the end of the paper.

2) In this paper the indices h, i, j, k, l, m run over the range $1, \cdots, 2n+1$; $\alpha, \beta, \gamma, \lambda, \mu$ the range $1, \cdots, n$; a, b, c, d the range $1, \cdots, 2n$; and A, B, C, D the range $1, \cdots, 2n+2$.