PRINCIPAL COFIBRATIONS

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Introduction. In this paper we shall define the notion of a principal cofibration which generalizes the cofibration $X \rightarrow C_f \rightarrow \Sigma Y$ induced by a map $f: X \rightarrow Y$ where C_f means the mapping cone of f and ΣY the reduced suspension of Y. The notion of a principal cofibration is a dual of a principal fibration in the sense of Peterson-Thomas [6].

One of the problem considered here is the following; under what conditions is a cofibration equivalent to a principal? This is answered by Theorem 2.7 in §2.

In §3 we dualize the results in [5], which are in the special case of induced cofibration. In §4 we mention an application to the Lusternik-Schnirelmann category and obtain a generalization of Berstein-Hilton's results.

1. Preliminaries. In this paper we assume that all spaces have base point denoted by $*$ and all maps (homotopies) preserve (keep fixed) base point.

A map $q: B \rightarrow E$ is called a cofibration if it has the homotopy lowering property for all spaces, i.e. if, for each space P and for all maps $f_0: E \rightarrow P$ and homotopies $g_t: B \rightarrow P$ with $g_0 = f_0 q$, there exists a homotopy $f_t: E \rightarrow P$ with $g_t = f_t q$. If q is an inclusion map, this is the homotopy extension property. The quotient space $F = E/q(B)$ is called the cofibre of q. Frequently the cofibration $q: B \rightarrow E$ with cofibre F will be denoted by the sequence $B \xrightarrow{q} E \xrightarrow{p} F$, where $p: E \rightarrow F$ is the projection.

Given a map $f: A \rightarrow B$, let C_f be the mapping cone of f, the space obtained from $CA \cup B$ by identifying $(a, 1) \in CA$ with $f(a)$, where CA denotes the reduced cone over A.

The set of all homotopy classes of maps $A \rightarrow B$ will be denoted by $\pi(A, B)$, which contains the distinguished element o, i.e. the homotopy class of the constant map $*: A \rightarrow B$. The homotopy class of a map $f: A \rightarrow B$ is denoted by $[f]$.

For maps $f: A \rightarrow C$ and $g: B \rightarrow C$, we define a map $f \bigtriangledown g: A \vee B \rightarrow C$ by

$$(f \bigtriangledown g)(a, *) = f(a) \qquad a \in A$$

$$(f \bigtriangledown g)(*, b) = g(b) \qquad b \in B$$