IMMERSIONS AND EMBEDDINGS OF TANGENT BUNDLES

FUICHI UCHIDA

(Received March 1, 1965)

Introduction. Let M^n be an n-dimensional compact connected differentiable manifold without boundary. Let $\tau(M^n) = (T(M^n), \pi, M^n)$ be the tangent bundle of M^n, where $T(M^n)$ is the total space and $\pi : T(M^n) \to M^n$ is the projection, then $T(M^n)$ is a $2n$-dimensional connected differentiable manifold.

In this paper we consider embeddings and immersions of $T(M^n)$ in Euclidean spaces. For example, in general $M^n \subset R^{2n}$, so $T(M^n) \subset R^{4n}$, but we get $T(M^n) \subset R^{3n}$. $M^n \subseteq R^{n+k}$ does not always imply $M^n \subset R^{n+k}$, but if $k > 0$ then for any $T(M^n)$, $T(M^n) \subseteq R^{2n+k}$ implies $T(M^n) \subset R^{2n+k}$.

In section 1, we recall some results of $KO(X)$ and immersion. In sections 2, 3 and 4, we prove some general theorems for immersions and embeddings of $T(M^n)$. And in sections 5 and 6, we consider several applications for the tangent bundles of projective spaces.

Notations $M \subset N$ and $M \subseteq N$ mean "M is differentiably embedded in N" and "M is differentiably immersed in N" respectively. $M \not\subset N$ and $M \not\subseteq N$ means "$M \subset N$ is false" and "$M \subseteq N$ is false" respectively.

1. Known results for $KO(X)$. Let X be a finite connected CW-complex and $KO_{(k)}(X)$ be the set of isomorphism classes of k-dimensional real vector bundles over X. Let $\mathcal{E}(X)$ denote the set of all isomorphism classes of real vector bundles on X. Then in our notation

$$\mathcal{E}(X) = \bigcup_{k=0}^{\infty} KO_{(k)}(X) .$$

$\mathcal{E}(X)$ is an abelian semigroup with zero for the Whitney sum. We shall use the symbol ξ^k to denote a k-dimensional vector bundle and ε^k to denote the trivial bundle of dimension k. We have a mapping

$$i_k : KO_{(k)}(X) \to KO_{(k+1)}(X)$$

defined by $i_k(\xi^k) = \xi^k \oplus \varepsilon^1$, and denote $\widetilde{KO}(X)$ the direct limit of this sequence and denote ξ_0 the corresponding class of ξ^k.

Now we can define the Grothendieck ring of real vector bundles over X