THE HOMOLOGY DECOMPOSITION FOR A COFIBRATION

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1. Introduction. As a homology analogue of the Postnikov decomposition, the homology decomposition of a 1-connected polyhedron was introduced by B. Eckmann and P. J. Hilton ([3], [5]). Moreover, as a generalization of this notion, B. Eckmann and P. J. Hilton ([4], [5]) and J. C. Moore ([6]) introduced the notion of the homology decomposition of a map. However, the homology decomposition of a map seems to be inconvenient for the actual applications.

Now as an intermediate notion of the above two decompositions, we introduce a notion of the homology decomposition for a cofibration $B \xrightarrow{q} X \xrightarrow{p} F$. This notion corresponds with a homology analogue of the Moore-Postnikov decomposition ([1]) and seems to have many applications in the algebraic topology.

In §3, we shall give a definition of the homology decomposition for a cofibration $B \xrightarrow{q} X \xrightarrow{p} F$ and its actual construction. If B reduces to a point, then such decomposition reduces to the usual homology decomposition for X. The decomposition for the cofibre F in such decomposition gives the usual one for F.

In §4 we introduce the notion of weak H'-cofibration as a generalization of the induced cofibration. The weak H'-cofibration weakens the notion of H'-cofibration defined in [7]. In §4, we explain the relations between the weak H'-cofibration and the homology decomposition for a cofibration.

2. Preliminaries. All spaces have base points denoted by $*$ and respected by maps $f, g, \cdots$ and their homotopies $f, g, \cdots$. Let $\pi(X, Y)$ denote the set of all homotopy classes of maps $X \to Y$. The homotopy class of a map $f: X \to Y$ is denoted by $[f]$. Let $K'(G, n)$ be a polyhedron with abelian fundamental group such that $H_r(K'(G, n))=0$ for $r \neq n$ and $H_n(K'(G, n))=G$. The homotopy type of the polyhedron $K'(G, n)$ is uniquely determined for $n \geqslant 2$. $K'(G, n)$ $(n \geqslant 2)$ has an H'-structure and we define the n-th homotopy group of X with coefficients in G by $\pi_n(G, X) = \pi(K'(G, n), X)$ and the n-th homotopy group of a map $f: X \to Y$ with coefficients in G by $\pi_n(G, f) = \pi_1(K'(G, n-1), f)$ ([2], [5]). Let $B \xrightarrow{q} X \xrightarrow{p} F$ be a cofibration and let $f: Y \to X$ be a map. Let C_f (resp. C_{pf}) denotes the space obtained by attaching the