ON CLASS NUMBERS OF FINITE ALGEBRAIC NUMBER FIELDS

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In this paper, we shall investigate some relations between the class number, the absolute ideal class group of a finite algebraic number field and that of its Galois extension of finite degree.

It is well known that the class number of the cyclotomic field is divisible by the class number of its subfield. In §1 we shall show that the class number of a finite algebraic number field is a divisor of the class number of its finite extension if the class number of the finite algebraic number field is prime to the degree of the extension field. In §2 we shall give main results of this paper which contain, as a special case, theorems of H. Weber, K. Iwasawa and others. In §3 we shall give a note on prime factors of the class number of the splitting field of a binomial equation with respect to the rational number field.

Throughout this paper the following notations will be used.

l, q : fixed rational prime numbers. p : any rational prime number.
$f_{q,p}$: the smallest positive integer f such that $q \mid p^f - 1$.
P : the rational number field.
$P_{(n)}$: the cyclotomic field generated by the primitive l^{n+1}-th root of unity over P.
k : the ground field which is a finite algebraic number field.
h_k : the class number of k. $h_{k,p}$: the p-part of h_k.
$V_{p,k}$: the least number of generators of the p-class group of k.
$[K:k]$: the relative degree of an extension K/k.
$\overline{k}_{(p)}$: the intermediate field of k and the absolute class field of k such that $[\overline{k}_{(p)}:k] = h_{k,p}$.
$G(K/k)$: the Galois group of a Galois extension K/k.

1. Let p be any prime number. The Sylow p-subgroup of the absolute ideal class group of k will be called *the p-class group of k.* Let K/k be a Galois extension with the Galois group $\mathfrak{g} = G(K/k)$. The Galois group $\mathfrak{g}$ acts on the ideal group of K and the p-class group of K in an obvious way. They may be considered as $\mathfrak{g}$-groups.

Let $\mathfrak{N}$ be the subgroup of all ideals $\mathfrak{a}$ in the ideal group $\mathfrak{D}$ of K such