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SOME NOTES ON THE GROUP OF AUTOMORPHISMS OF CONTACT AND SYMPLECTIC STRUCTURES

YOJI HATAKEYAMA[1)]

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Introduction. In this paper, we shall prove that the group of automorphisms of a contact or a symplectic structure defined on a compact manifold M^n acts transitively on it. For this purpose, we first notice that we can find a linear mapping h from the linear space of differentiable functions $\mathfrak{F}$ defined on M^n to that of infinitesimal automorphisms $\boldsymbol{L}$ of these structures; these mappings were introduced by J.W.Gray and P.Libermann in the case of a contact structure and a symplectic structure respectively, and many properties of h were studied by them, (cf. [3][2)] and [4]), but in this paper, we only need the fact that for any differentiable function ρ over M^n, $h(\rho)$ gives an infinitesimal automorphism of these structures. Next, we consider n functions defined over M^n which give a canonical coordinate system around a point $P \in M^n$ associated with such structures, and making use of these functions and the mapping h, we shall prove our theorems.

1. The transitivity of the group of automorphisms of a contact structure. Let $M^n (n=2m+1)$ be a differentiable manifold with a contact structure defined by a 1-form η, i.e., let M^n admit a 1-form η satisfying the relation

$$\eta \wedge \overbrace{d\eta \wedge \cdots \wedge d\eta}^{m} \neq 0, \tag{1.1}$$

where $d\eta$ and $\wedge$ mean the exterior derivative of η and exterior product respectively. Then, we can find a uniquely determined vector field ξ defined over M^n which satisfies the relations

$$i(\xi)\eta = 1 \text{ and } i(\xi)d\eta = 0, \tag{1.2}$$

1) The material of this work is a section of thesis for the degree of Doctor of Science in Tôhoku University, 1965.
2) Numbers in brackets refer to the bibliography at the end of the paper.