Tôhoku Math. Journ.
Vol. 19, No. 2, 1967

ON PROPERTY P OF VON NEUMANN ALGEBRAS

JÔSUKE HAKEDA

(Received February 20, 1967)

Many results are known for the unitary invariants of von Neumann algebras, but, unfortunately, very little is known for algebraic one, especially for continuous von Neumann algebras, even in the case of factors. Recently, J.Schwartz has shown the existence of the new factor of type II_1 ([1]), by introducing a new property, called property P. The definition of property P is not given in the algebraic form, however, it is very suggestive that the isomorphic images of known factors with property P have also property P. The purpose of this paper is to show that property P is actually an algebraic invariant, i.e. invariant under isomorphism.

1. We shall start with our key lemma in rather general form. Let E be a Banach space which is the conjugate space of a Banach space and G a uniformly bounded group of continuous linear transformations on E with respect to the weak* topology. Let E_0 be the subset of E whose elements are G-invariant, i.e. $E_0=\{x\in E: sx=x \text{ for all } s\in G\}$. By $\mathfrak{C}$, we shall denote the set of all non-negative real valued function f on G such that $f(s)=0$ for $s\in G$ with finite numbers of exceptions and $\sum_{s\in G} f(s)=1$. $\widetilde{f}(x)$ will be defined as follows: $\widetilde{f}(x)=\sum_{s\in G} f(s)sx$. For arbitrary $x\in E$, K_x will denote the closure of the set of all $\widetilde{f}(x)$ for $f\in\mathfrak{C}$ with respect to the weak* topology. Then K_x is compact with respect to the weak* topology. Hereafter, we use the weak* topology in the argument on E without any proviso.

LEMMA 1.1. *If there exists a directed sequence $(f_\alpha)_{\alpha\in A}\subset\mathfrak{C}$ such that $\widetilde{f}_\alpha(x)$ converges to some $\tau(x)\in K_x$ for every $x\in E$, then, for arbitrary $h\in\mathfrak{C}$, there exists a directed sequence $(h_\alpha)_{\alpha\in A}\subset\mathfrak{C}$ such that $\widetilde{h}_\alpha(x)$ converges to $\widetilde{h}(\tau(x))$ for every $x\in E$.*

PROOF. We define the directed sequence $(h_\alpha)_{\alpha\in A}\subset\mathfrak{C}$ as follows:

$$h_\alpha(r)=\sum_{s\in G} h(s)f_\alpha(s^{-1}r) \quad \text{for} \quad r\in G.$$

By the continuity of every elment of G, $\sum_{r\in G} f_\alpha(s^{-1}r)rx$ converges to $s\tau(x)\in K_x$.