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ON A CONFORMALLY FLAT RIEMANNIAN SPACE WITH POSITIVE RICCI CURVATURE

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1. Introduction. A Riemannian space is called conformally flat when its Weyl conformal curvature tensor vanishes. It is well known that the p-th $(0 < p < n)$ Betti numbers of a compact orientable conformally flat Riemannian space with positive Ricci curvature are all zero.[1] In this paper we shall prove the following theorems in the case when its scalar curvature is constant.

THEOREM A. *Let M be a compact orientable conformally flat Riemannian space with constant scalar curvature and with positive Ricci curvature, then M is of constant curvature.*

In the proof of this theorem, we can find that the same conclusion is obtained by replacing the condition about the Ricci curvature by that of positive sectional curvature. Hence we have:

THEOREM B. *Let M be a compact orientable conformally flat Riemannian space with constant scalar curvature and with positive sectional curvature, then M is of constant curvature.*

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2. Definitions and notations. Let M be an n-dimensional Riemannian space and p be a point in M. $T_p(M)$ and $\{x^a\}$ denote the tangent space to M at p and a local coordinate system around p, respectively. As usual, g_{ab}, $R^a{}_{bcd}$ and $R_{bc} = R^a{}_{bca}$ denote the Riemannian metric, the curvature tensor and the Ricci tensor, respectively. Take an orthonormal basis $X_1, \cdots, X_n$ at p, then we have

$$g_{ab}\, X_i{}^a\, X_j{}^b = \delta_{ij},$$

1) K. Yano and S. Bochner, [1], pp. 79–80.