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APPLICATIONS OF FUBINI TYPE THEOREM TO THE TENSOR PRODUCTS OF C*-ALGEBRAS

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Let A and B be C^*-algebras and $A \underset{\alpha}{\otimes} B$ their C^*-tensor product with α-norm ([16]). We consider the following linear mapping from $A \underset{\alpha}{\otimes} B$ to A and B defined by

$$R_\varphi\left(\sum_{i=1}^{n} a_i \otimes b_i\right) = \sum_{i=1}^{n} < a_i, \varphi > b_i \quad \left(\text{resp. } L_\psi\left(\sum_{i=1}^{n} a_i \otimes b_i\right) = \sum_{i=1}^{n} < b_i, \psi > a_i\right)$$

for a bounded functional φ of A (resp. ψ of B). This mapping satisfies the following relation

$$< x, \varphi \otimes \psi > = < L_\psi(x), \varphi > = < R_\varphi(x), \psi >$$

for every $x \in A \underset{\alpha}{\otimes} B$.

Now the above relation may be considered, in some sense, as the non-commutative version of Fubini theorem in iterated integrals and it is the purpose of our present discussions to clarify the utility of this result in the tensor products of C^*-algebras settling all type problems of product algebras (Theorem 2) by using this mapping, and deriving various structure theorems for them, some of which are regarded as the extension of several results in [7], [19].

The above mapping is also useful to more general situations (cf. [10], [15]), since it can be defined in any tensor product of Banach algebras whenever the defining cross-norm is not less than Schatten's λ-norm ([13]).

Through the discussions $S(A)$ means the set of all states of a C^*-algebra A and $P(A)$ means the set of all pure states of A. The value of a linear functional φ on x is always denoted as $< x, \varphi >$. Let $A \odot B$ be algebraic tensor product of A and B, then the norm α is given by