Tôhoku Math. Journ.
Vol. 19, No. 2, 1967

ON A DECOMPOSITION OF C-HARMONIC FORMS IN A COMPACT SASAKIAN SPACE

SHUN-ICHI TACHIBANA

(Received January 16, 1967)

0. Introduction. Let M be a compact regular Sasakian space, $\pi: M \to B$ the fibering of M. Recently S. Tanno [10] discussed relations between the Betti numbers of M and B by making use of the exact sequence of Gysin. On the other hand it is well known that any harmonic p-form $(p \leqq m+1)$ in a compact Kählerian space M^{2m} is written in terms of effective harmonic forms and the fundamental 2-form of M^{2m}. The work by Tanno suggests that an analogous theorem is expected in a compact Sasakian space.

In this paper, first we fix our notations in §1 and introduce a notion of a C-harmonic form in a compact Sasakian space in §4. The decomposition theorem for C-harmonic form will be given in the last section. We shall give only outline of proofs by the following two reasons: (1) the discussions in §2 and §5 are similar to that of an almost Hermitian space and a Kählerian space, (2) the results in §4 are based on straightforward computations though they are rather complicated and it is expected to have a reformulation by Y. Ogawa in a forthcoming paper [4].

1. Preliminaries.[1] Consider an n dimensional Riemannian space M^n and let $\{x^\lambda\}$, $\lambda = 1, \cdots, n$, be its local coordinates. Denoting the positive definite Riemannian metric by $g_{\lambda\mu}$, the Riemannian curvature tensor and the Ricci tensor are given by

$$R_{\lambda\mu\nu}{}^{\omega} = \partial_\lambda \begin{Bmatrix} \omega \\ \mu\nu \end{Bmatrix} - \partial_\mu \begin{Bmatrix} \omega \\ \lambda\nu \end{Bmatrix} + \begin{Bmatrix} \omega \\ \lambda\alpha \end{Bmatrix} \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} - \begin{Bmatrix} \omega \\ \mu\alpha \end{Bmatrix} \begin{Bmatrix} \alpha \\ \lambda\nu \end{Bmatrix},$$

$$R_{\mu\nu} = R_{\varepsilon\mu\nu}{}^{\varepsilon},$$

where $\begin{Bmatrix} \nu \\ \lambda\mu \end{Bmatrix}$ means the Christoffel symbol and $\partial_\lambda = \partial/\partial x^\lambda$.

Components of a skew-symmetric tensor $u_{\lambda_1 \cdots \lambda_p}$ are considered as coefficients of a differential form:

1) As to notations we follow S. Tachibana [8].