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NOTES ON COVARIANT ALMOST ANALYTIC VECTOR FIELDS

KENTARO YANO AND MITSUE AKO

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1. Introduction. In a previous paper [4] we defined almost analytic vector fields in an almost complex space and generalized some of well known results for analytic vector fields in a Kähler space to those for almost analytic vector fields in the most general almost Hermitian space.

To define a contravariant almost analytic vector field we proceeded as follows :

In a complex manifold M covered by a system of neighborhoods U with complex coordinates $(z^\kappa, z^{\bar\kappa})$[1], a self conjugate contravariant vector field $(v^\kappa, v^{\bar\kappa})$, that is, a contravariant vector field $(v^\kappa, v^{\bar\kappa})$ satisfying $\overline{v^\kappa} = v^{\bar\kappa}$, is said to be analytic when the components v^κ and $v^{\bar\kappa}$ are analytic functions of z and $\bar z$ respectively :

$$v^\kappa = v^\kappa(z), \quad v^{\bar\kappa} = v^{\bar\kappa}(\bar z). \tag{1.1}$$

The condition (1.1) is equivalent to

$$\partial_{\bar\lambda} v^\kappa = 0, \quad \partial_\lambda v^{\bar\kappa} = 0, \tag{1.2}$$

where $\partial_{\bar\lambda}$ means $\partial/\partial z^{\bar\lambda}$ and ∂_λ means $\partial/\partial z^\lambda$.

On the other hand, we have, in a complex manifold, a numerical tensor F of type (1, 1) given by

$$F_i^{\ h} = \begin{pmatrix} \sqrt{-1}\,\delta_\lambda^\kappa & 0 \\ 0 & -\sqrt{-1}\,\delta_{\bar\lambda}^{\bar\kappa} \end{pmatrix}^{2)} \tag{1.3}$$

and consequently, putting

1) Here and in the sequel the Greek indices $\kappa, \lambda, \mu, \cdots$ run over the range $\{1, 2, \cdots, n\}$ and $\bar\kappa, \bar\lambda, \bar\mu, \cdots$ the range $\{\bar 1, \bar 2, , \cdots, \bar n\}$.

2) Here and in the sequel, the Roman indices $h, i, j, \cdots$ run over the range $1, 2, \cdots, n$; $\bar 1, \bar 2, \cdots, \bar n$.