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ON A CLASS OF CONVOLUTION TRANSFORM II

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1. Introduction. In the preceding paper [7], we studied the inversion theory for some restricted class of convolution transform

$$(1)\qquad f(x)=\int_{-\infty}^{\infty}G(x-t)\,e^{ct}\,d\alpha(t)\qquad (c:\text{real}),$$

for which the kernel $G(t)$ is of the form

$$(2)\qquad G(t)=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}[F(s)]^{-1}\,e^{st}\,ds\,.$$

Here $F(s)$ is the meromorphic function with real zeros and poles only, and is of the form $F(s)=E_1(s)/E_2(s)$,

$$(3)\qquad E_1(s)=e^{bs}\prod_{1}^{\infty}(1-s/a_k)\,e^{s/a_k},\quad E_2(s)=\prod_{1}^{\infty}(1-s/c_k)\,e^{s/c_k},$$

where b, $\{a_k\}_1^\infty$, $\{c_k\}_1^\infty$ are constants such that $\sum_1^\infty a_k^{-2}<\infty$, $\sum_1^\infty c_k^{-2}<\infty$.

When $E_1(s)$ and $E_2(s)$ are reciprocals of the generating functions of kernels of class I and class II (or I, II) [3], [4], respectively, and $F(s)$ satisfies some conditions, we knew that the properties of the transform (1) are similar to those of the convolution transform with the class I kernel.

In this paper we shall study the case in which both $E_1(s)$ and $E_2(s)$ are reciprocals of the generating functions of class II (or III) kernels and satisfies the conditions:

$$(4\text{a})\qquad a_kc_k>0\,,\quad |a_k|\leqq|c_k|\quad\text{for all } k \text{ and } \left|\sum_1^\infty(a_k^{-1}-c_k^{-1})\right|<\infty\,;$$

(4 b) for some positive α and any positive number R,