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ON CONTRACTION OF WALSH FOURIER SERIES

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The purpose of this note is to prove the Walsh analogue of results due to M. Kinukawa [3] and an extension of the theorem on contraction of C. Watari [4].

We begin with some notations and definitions:

Following A. Beurling, $g(x)$ is called a contraction of $f(x)$ if

$$|g(x)-g(x')| \leqq |f(x)-f(x')| \quad \text{for} \quad x, x' \in (0,1).$$

A sequence $\{a_n\}$ is called a contraction of sequence $\{c_n\}$ if

$$|a_m-a_n| \leqq |c_m-c_n| \quad \text{for every } m \text{ and } n.$$

The Rademacher functions are defined by

$$\phi_0(x) = 1 \quad (0 \leqq x < 1/2), \qquad \phi_0(x) = -1 \quad (1/2 \leqq x < 1)$$
$$\phi_0(x) = \phi_0(x+1), \qquad \phi_n(x) = \phi_0(2^n \cdot x) \quad (n = 1, 2, \cdots).$$

The Walsh functions are then given by

$$\psi_0(x) \equiv 1, \quad \psi_n(x) = \psi_{n(1)}(x)\psi_{n(2)}(x)\cdots\psi_{n(r)}(x),$$

for $n=2^{n(1)}+2^{n(2)}+\cdots+2^{n(r)} \geqq 1$, where the integers $n(i)$ are uniquely determined by $n(i+1)<n(i)$. For basic properties of Walsh functions, the reader is referred to N. J. Fine [2]. Finally, A denotes a positive absolute constant not always the same. The author wishes to express his hearty thanks to Prof. C. Watari for his valuable suggestions and encouragements in the preparation of this paper. The author also thanks Prof. S. Igari for better presentation.

Our results are as follows:

THEOREM 1. *Let*