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WEAKENED BERTRAND CURVES

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1. Introduction. In the discussion of Bertrand curves (in fact nearly all curves) in elementary classical differential geometry, it is always assumed (explicitly or implicitly) that the curvature is nowhere zero. In this paper we drop this requirement on the Bertrand curves and investigate into the properties of two types of similar curves (the Frenet-Bertrand curves and the weakened Bertrand curves) under weakened conditions. The properties of the Frenet-Bertrand curves turn out to be strikingly similar to those of the Bertrand curves.

We first put down our convention of defining regular curves. We also use the convention that if f is a real function on a set A, by $f=0$ we mean that f is everywhere zero on A, and by $f\neq 0$ we mean that f is nowhere zero on A.

DEFINITION 1.1. A *parametrized curve* (simply called a *curve*) in E^3 is a point set Γ in E^3 together with an equivalence class of continuous and locally injective surjections

$$(\phi, \psi, \chi): L \rightarrow \Gamma$$

defined by

$$(\phi, \psi, \chi)(t) = (\phi(t), \psi(t), \chi(t)),$$

where two such mappings (ϕ, ψ, χ), $(\widetilde{\phi}, \widetilde{\psi}, \widetilde{\chi})$ which are defined on two intervals L, $\widetilde{L}$ respectively are said to be equivalent if there exists a continuous, strictly monotonic increasing function

$$\sigma: L \rightarrow \widetilde{L}$$

with $\widetilde{L}$ as image set (σ must then be a bijection from L onto $\widetilde{L}$) such that

$$(\phi, \psi, \chi) = (\widetilde{\phi}, \widetilde{\psi}, \widetilde{\chi}) \circ \sigma .$$

Each of the mappings (ϕ, ψ, χ) is called a *parametrization* of Γ.