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ON THE EXISTENCE OF O-CURVES II *)

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Cooke [1] has discussed asymptotic behaviors of solutions of a functional differential equation

$$(1) \qquad \dot{u}(t) + au(t - r(t)) = 0$$

under the assumption that $r(t)$ is a non-negative continuous function which satisfies the conditions

$$r(t) \to 0 \quad \text{as} \quad t \to \infty \quad \text{and} \quad \int_0^\infty r(t)\,dt < \infty .$$

In the previous paper [3], we have obtained some results concerning the existence of O-curves and some kind of the asymptotic equivalence, which we shall call the asymptotic semi-equivalence (for the definition, see the below). By applying the similar arguments to those used in [3], we shall discuss the same problems as discussed by Cooke, for more general equations.

Here, we shall give the following definitions:

DEFINITION 1. A solution of a system will be called to be *an O-curve* of the system, if it tends to zero as $t \to \infty$.

DEFINITION 2. Two systems (E_1) and (E_2) are said to be *asymptotically semi-equivalent*, provided that for any bounded solution of (E_1) (or (E_2)) we can find a solution of (E_2) (or (E_1)) which approaches the bounded solution of (E_1) (or (E_2), respectively) for infinitely increasing t. In the case where we can remove the boundedness for the given solution, two systems (E_1) and (E_2) are *asymptotically equivalent* (cf. [2]).

Let $r \geqq 0$ be a given constant. C^n denotes the space of continuous functions mapping the interval $[-r, 0]$ into the Euclidean n-space E^n with a norm $\|\varphi\|_r$ defined by

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