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# A TAUBERIAN CONSTANT FOR THE $(S, \mu_{n+1})$ TRANSFORMATION

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**1.** Corresponding to any fixed sequence $\{\mu_n\}$, Ramanujan [17] introduced the summability method given by the sequence-to-sequence transformation

$$t_n = \sum_{k=0}^{\infty} \binom{n+k}{n} (\Delta^k \mu_{n+1})\, s_k, \tag{1.1}$$

where

$$\Delta\mu_n = \mu_n - \mu_{n+1},\ \Delta^0\mu_n = \mu_n,\ \Delta^k\mu_n = \Delta(\Delta^{k-1}\mu_n). \tag{1.2}$$

Writing

$$t_n = b_0 + b_1 + \cdots + b_n;\ s_k = a_0 + a_1 + \cdots + a_k, \tag{1.3}$$

we shall see in §2 that (1.1) is formally the same as

$$\begin{cases} b_0 = \sum_{k=0}^{\infty} (\Delta^k \mu_0)\, a_k, \\ b_n = \sum_{k=1}^{\infty} \binom{n+k-1}{n} (\Delta^k \mu_n)\, a_k, \qquad (n \geqq 1). \end{cases} \tag{1.4}$$

We shall also see that for those sequences for which, for every fixed $n$

$$\binom{n+k-1}{n} (\Delta^k \mu_n) s_k \to 0, \qquad \text{as} \quad k \to \infty, \tag{1.5}$$

(1.1) and (1.4) are, in fact, equivalent in the following sense. Suppose that (1.5) holds. Then, if (1.1) converges for all $n$, (1.4) holds; and if (1.4) converges for all $n$, (1.1) holds.

It will be convenient to change Ramanujan's notation slightly, and to denote the summability method given by the series-to-series transformation (1.4) by