Tôhoku Math. Journ.
Vol. 19, No. 2, 1967

ON 5-DIMENSIONAL SASAKI-EINSTEIN SPACE WITH SECTIONAL CURVATURE $\geqq 1/3$

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(Received July 23, 1966)

1. Introduction. S. I. Goldberg has proved the following theorems [4];

THEOREM. *If a compact, simply connected regular Sasakian space*[1] *has positive sectional curvature and constant scalar curvature, then it is isometric to a Euclidean sphere of the same dimension.*

THEOREM. *If a compact regular Sasakian space has positive sectional curvature, then its second Betti number vanishes.*

For the proofs of these theorems, the assumption of regularity of the contact structure is inevitable. Without the assumption of regularity of the first theorem we have proved the following [6]

THEOREM. *If a complete $2m+1(\geqq 5)$-dimensional Sasakian space has sectional curvature $> 1/2m$, then the second Betti number vanishes.*

On the other hand, M. Berger proved the following [3]

THEOREM. *If a complete, Kähler-Einstein space has positive sectional curvature, then it is isometric to a complex projective space with a metric of constant holomorphic sectional curvature.*

In a former paper [2], he has proved the following theorem as a special case of this theorem.

If a 4-dimensional compact Kähler-Einstein space has non-negative sectional curvature, then it is a locally symmetric space.

To exclude regularity condition of the second theorem of Goldberg, we apply the Berger's method to 5-dimensional Sasaki-Einstein space and obtain

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1) In this note, manifolds are assumed to be connected and C^∞-differentiable.