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ON SOME EXTENTION PROPERTIES OF VON NEUMANN ALGEBRAS

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1. In the theory of the structure of von Neumann algebras, a great deal of discussions has been devoted to the algebraic invariants. Recently, J. Schwartz [6], has shown a new behavior of the hyperfinite factor by introducing a new property, called property P, and proved the existence of new algebraic type of continuous finite factor. Very recently, one of the authors has proved that property P is an algebraic invariant [2] and it makes us to have some interest that the main results of the paper [6], especially the key results [6: Cor. 6 and Lemma 7], can be deduced only from the point of view of the existence of a projection mapping of norm one.

Thus we shall investigate in the following the algebraic version of property P as an extension property of the commutant of a given von Neumann algebra. We shall also study this extension property as the property of the commutant itself. These properties will turn out to be algebraic invariants and it is proved that our extension property can be defined space-freely in a form very similar to the usual extension property of Banach space. Relationships between tensor products of von Neumann algebras and these properties are also studied.

2. In the following we denote by $\mathfrak{L}(\mathfrak{H})$, the algebra of all bounded linear operators on a Hilbert space $\mathfrak{H}$. We shall define the extension property of a von Neumann algebra $\mathfrak{A}$ as follows.

DEFINITION 2.1. A von Neumann algebra $\mathfrak{A}$, acting on a Hilbert space $\mathfrak{H}$, has extension property, if there exists a projection of norm one from $\mathfrak{L}(\mathfrak{H})$ to $\mathfrak{A}$.

If we consider this extension property as the property of the commutant $\mathfrak{A}'$, of $\mathfrak{A}$, we get an algebraic version of the property P in Schwartz [6].

DEFINITION 2.2. A von Neumann algebra $\mathfrak{A}$ is called to have abstract