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ON THE CONVOLUTION ALGEBRA OF BEURLING

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1. Introduction. Let $f(x)$ be an integrable function with period 2π and its Fourier series be

$$S(f) = \sum_{n=-\infty}^{\infty} c_n e^{inx}.$$

We write $\widehat{A}$ for the class of functions with absolute convergent Fourier series. $\widehat{A}$ is a Banach algebra under usual operations. In this algebra, spectral synthesis is impossible and operating functions are analytic. A. Beurling [1] considered a subclass of $\widehat{A}$ such that

$$A_0 = \{f \mid A(f) < \infty\}$$

where

$$A(f) = \int_0^1 t^{-3/2} \left\{ \int_0^{2\pi} |f(x+t) - f(x-t)|^2 \, dx \right\}^{1/2} dt .$$

The algebra A_0 has remarkable properties, that is to say, that spectral synthesis is possible and the functions which satisfy the Lipschitz condition of order 1 are operating.

In this note, we extend slightly $A(f)$ to

$$(1) \qquad A_\beta(f) = \int_0^1 t^{-2+\beta/2} \left\{ \int_0^{2\pi} |f(x+t) - f(x-t)|^2 \, dx \right\}^{\beta/2} dt$$

for $1 \leqq \beta < 2$ and show that $A_\beta(f) < \infty$ is equivalent to $B_\beta(f) < \infty$ or $C_\beta(f) < \infty$, where

$$(2) \qquad B_\beta(f) = \sum_{n=1}^{\infty} n^{-\beta/2} \left\{ \sum_{|k|=n+1}^{\infty} |c_k|^2 \right\}^{\beta/2}$$