Tôhoku Math. Journ.
Vol. 19, No. 3, 1967

ON C-HARMONIC FORMS IN A COMPACT SASAKIAN SPACE

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(Received March 6, 1967)

Introduction. It is well known that in a $2m$-dimensional compact Kählerian space, any harmonic p-form $(p \leqq m)$ can be written uniquely in terms of effective harmonic forms and the fundamental 2-form of the space. When we consider the analogy in a compact Sasakian space, it is insignificant as far as we are concerned about harmonic forms, because any harmonic form is effective. S. Tachibana [1] has introduced the notion of C-harmonic forms in a compact Sasakian space, which is wider than that of harmonic forms, and succeeded to prove the analogy of the decomposition theorem for C-harmonic forms. In this paper we try to make the definition of C-harmonic forms a little looser than that of Tachibana's original one. On the other hand, S. Tanno has drawn the relation of Betti numbers between the base space and the bundle space in the fibering of a regular K-contact Riemannian space. It is shown that a p-form $(p \leqq m)$ on the bundle space is C-harmonic if and only if it is induced from a harmonic p-form on the base space. Thus we can obtain the theorem of Tanno again. Lastly we investigate the C^*-harmonic forms which are dual to the C-harmonic forms, and in connection with them, we observe Killing forms and give one of its example on a Sasakian space.

Manifolds are assumed to be connected and the differentiable structures on them are assumed to be of class C^∞.

I should like to express my hearty gratitude to Prof. S. Tachibana for his kind suggestions and many valuable advices.

Contentes are as follows:

1. Preliminaries. An n-dimensional Riemannian space M^n is called a Sasakian space if it admits a unit Killing vector field η^λ such that

$$\nabla_\lambda \nabla_\mu \eta_\nu = \eta_\mu g_{\lambda\nu} - \eta_\nu g_{\lambda\mu}, \tag{1.1}$$