Tôhoku Math. Journ.
Vol. 19, No. 3, 1967

ON PARALLEL HYPERSURFACES OF AN ELLIPTIC HYPERSURFACE OF THE SECOND ORDER IN E^{n+1}

TOMINOSUKE ŌTSUKI

(Received February 10, 1967)

In [1] M. Berger stated a theorem which is equivalent to the following:

Let M be a complete Riemannian manifold whose sectional curvature K satisfies the inequality

$$(1) \qquad 0 < A \leqq K(\Pi) \leqq B,$$

where A and B are positive constants and Π is any tangent plane to M. Let X be any Jacobi field along a geodesic $x = \gamma(s)$ parameterized with arclength s such that

$$(2) \qquad \|X(0)\| = 1, \quad X'(0) = 0, \quad <X(0), \gamma'(0)> = 1,$$

then

$$(3) \qquad \|X(s)\| \leqq \cos\sqrt{A}\, s \quad \text{for} \quad 0 \leqq s \leqq \frac{\pi}{2\sqrt{B}}.$$

This statement is made sure of its truth in the case dim M=2 or M is locally symmetric, using their properties. Regarding this theorem, the author will investigate the curvature of the following elementary spaces which are generally non-symmetric.

An elliptic hypersurface Q of order 2:

$$(4) \qquad \sum_{\lambda=1}^{n+1} \frac{1}{a_\lambda^2} x_\lambda^2 = 1 \quad (a_1, \cdots, a_{n-1} > 0)^{1)}$$

in the $(n+1)$-dimensional Euclidean space E^{n+1} with the orthogonal coordinates $x_1, \cdots, x_{n+1}$, is, as well known, an n-dimensional compact Riemannian manifold with positive sectional curvature. The parallel hypersurface Q_c of Q which is the locus of the points with distance c from each point on

1) In this paper, Greek indices run from 1 to $n+1$ and Latin indices from 1 to n.