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OPERATING FUNCTIONS ON SOME SUBSPACES OF l_p

YOSHIKAZU UNO

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1. Let $L^2(0, 2\pi)$ be the set of all square integrable functions defined on $(0, 2\pi)$ and continued by periodicity. We set

$$A_{\beta,\delta}(f) = \left[\int_0^1 \frac{dt}{t^{2-\beta/2+\delta}}\left\{\int_0^{2\pi} |f(x+t) - f(x-t)|^2\, dx\right\}^{\beta/2}\right]^{1/\beta}$$

for $f \in L^2(0, 2\pi)$, where $1 \leqq \beta \leqq 2$ and $3\beta/2 - 1 > \delta > \beta/2 - 1$.

We define a space $A_{\beta,\delta}$ by

$$A_{\beta,\delta} = \{f\colon A_{\beta,\delta}(f) < \infty\}\,.$$

If $f \in A_{\beta,\delta}$ and $f_a(x) = f(x-a)$, then $A_{\beta,\delta}(f_a) = A_{\beta,\delta}(f)$, if c is a constant, then $A_{\beta,\delta}(cf) = |c|A_{\beta,\delta}(f)$ and if f, $g \in A_{\beta,\delta}$, then $A_{\beta,\delta}(f+g) \leqq A_{\beta,\delta}(f) + A_{\beta,\delta}(g)$ by Minkowski's inequality.

We shall characterize the complex valued function φ of a complex variable which operates in $A_{\beta,\delta}$ i.e. $\varphi(f) \in A_{\beta,\delta}$ for all $f \in A_{\beta,\delta}$, where $\varphi(f)(x) = \varphi(f(x))$.

2. Let the Fourier series of $f \in L^2(0, 2\pi)$ be

$$f(x) \sim \sum_{n=-\infty}^{\infty} c_n e^{inx}\,.$$

For β and δ which satisfy the above conditions, we set

$$B_{\beta,\delta}(f) = \left\{\sum_{n=1}^{\infty} n^{-\beta/2+\delta}\Big(\sum_{|k|>n} |c_k|^2\Big)^{\beta/2}\right\}^{1/\beta}$$

$$C_{\beta,\delta}(f) = \left\{\sum_{n=1}^{\infty} n^{-3\beta/2+\delta}\Big(\sum_{|k|\leqq n} |c_k|^2 k^2\Big)^{\beta/2}\right\}^{1/\beta}\,.$$