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COMPACT ORIENTABLE SUBMANIFOLD OF CODIMENSION 2 IN AN ODD DIMENSIONAL SPHERE

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Introduction. It has been proved by H. Liebmann [3] that the only ovaloid with constant mean curvature in Euclidean space E^3 is a sphere. The analogous theorem for a convex m-dimensional hypersurface in E^{m+1} has been proved by W. Süss [6]. Recently Y. Katsurada [1], [2] and K. Yano [9] have generalized the above theorem to an m-dimensional hypersurface in an Einstein space admitting one-parameter groups of conformal transformations or of homothetic transformations.

Thus we may expect an analogous theorem for a submanifold of codimension greater than 1 in a certain Riemannian manifold. On the other hand the present author studied, in the previous paper [4], a certain hypersurface in an odd dimensional sphere S^{2n+1} and found that the natural contact structure of S^{2n+1} plays an important role in the study of the hypersurface of S^{2n+1}.

This fact suggests that, using the natural contact structure of S^{2n+1}, we can solve the problem similar to the Liebmann-Süss problem for a submanifold of codimension 2 in an odd dimensional sphere.

The purpose of the paper is to prove the analogue of the Liebmann-Süss theorem for a submanifold of codimension 2 in S^{2n+1}. For this purpose, we give in §1, some properties of the contact structure of S^{2n+1} and in §2 some formulas in the theory of submanifold of codimension 2. In §3, we study a submanifold of codimension 2 in an odd dimensional sphere and introduce some quantities for later use.

In §4 some integral formulas for a submanifold of codimension 2 in an odd dimensional sphere are derived and under certain conditions the theorem mentioned above is proved. However an umbilical submanifold of codimension 2 in $(2n+1)$-dimensional sphere does not necessarily satisfy the conditions of our theorem. So in §5 we show an example of umbilical submanifold which satisfies our conditions.

1. Contact Riemannian structure on an odd dimensional sphere. A $(2n+1)$-dimensional differentiable manifold M is said to have a contact