Tôhoku Math. Journ.
20(1968), 582-588.

THE OPERATIONS ρ_R^k ON THE GROUP $\widetilde{K}_R(CP^n)$

KOICHI IWATA AND HIROSHI ÔIKE

(Received June 11, 1968)

The operations ρ_C^k and ρ_R^k play significant roles in K-theory. Their definitions and the actions on the K-groups of diverse complexes are described in [2].

In this note, we calculate the action of ρ_R^k on the reduced K-rings of a complex projective space CP^n. The method used here is the same as the one which is employed by J. F. Adams in the calculation of ρ_R^k on the ring $\widetilde{K}_R(S^{4n})$ [2. (5. 18)].

Preliminaries. Let CP^n be the (complex) n-dimensional complex projective space and $\widetilde{K}_C(CP^n)$ (resp. $\widetilde{K}_R(CP^n)$) be its complex (resp. real) (reduced) K-rings. We write

$$c: \widetilde{K}_R(CP^n) \longrightarrow \widetilde{K}_C(CP^n),$$

$$r: \widetilde{K}_C(CP^n) \longrightarrow \widetilde{K}_R(CP^n),$$

$$t: \widetilde{K}_C(CP^n) \longrightarrow \widetilde{K}_C(CP^n)$$

for the homomorphisms induced by complexification, realification and complex conjugation. As is well-known ([1], Lemma 3.9), we have

$$cr = 1 + t\,,$$

$$rc = 2\,.$$

The ring $\widetilde{K}_C(CP^n)$ is generated by one generator μ which satisfies the relation $\mu^{n+1}=0$ ([1], Theorem 7.2). The ring $\widetilde{K}_R(CP^n)$ is generated by one generator $\omega=r\mu$ which satisfies the following relations:

$$\omega^{2w+1} = 0\,, \qquad \text{if} \quad n = 4w\,,$$

$$2(\omega^{2w+1}) = 0\,, \quad \omega^{2w+2} = 0\,, \qquad \text{if} \quad n = 4w+1\,,$$