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STIFF GROUPS AND WILD SOCLES

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All groups in this paper are assumed to be p-primary abelian groups for a fixed prime p. We follow the terminology and, with minor exceptions, the notation of [4]. We refer to $G[p]$ as the *socle* of G and by a *subsocle* of G we shall mean a subgroup of $G[p]$. All topological references are to the p-adic topology. By a *dense subsocle* of G we shall mean a subgroup of $G[p]$ that is dense in the topology on $G[p]$ induced by the p-adic of G. G^1 will denote the subgroup of elements of infinite height in G, that is, $G^1 = \bigcap_{n<\omega} p^n G$. If $G^1=0$, G is contained as a pure, dense subgroup of a closed p-group K. The purity of G in K implies that the p-adic topology of K induces that of G and G being dense in K means that K/G is divisible. For a given G, K is unique up to isomorphisms leaving the elements of G fixed and will be referred to as the *torsion-completion* of G.

We shall let $E(G)$ denote the endomorphism ring of G. Following Crawley [2], G is said to be *stiff* if for each $\phi \in E(G)$ there is an $n < \omega$ such that $\phi|(p^nG)[p]$ is multiplication by an integer. A dense subsocle S of G will be called *wild* if whenever $\phi \in E(G)$ and $\phi(S) \subseteq S$ there exists an $n < \omega$ such that $\phi|(p^nG)[p]$ is multiplication by an integer. If $G^1=0$ and $G[p]$ is a wild subsocle of the torsion completion K of G, then we shall say that *G has a wild socle.* Since endomorphisms of G extend uniquely to endomorphisms of K, a group G is stiff if it has a wild socle.

It is easily seen that stiff groups are *essentially indecomposable* (that is, if G is stiff and if $G=A\oplus B$, then one of the two groups A and B is bounded). Crawley [2] has also shown that stiff groups have the *exchange property*. The first construction of a stiff group was by Crawley in [1] where he found a wild subsocle of the torsion-completion of $\bigoplus_{n<\omega} C(p^n)$. This first construction of a group having a wild socle was in connection with finding an infinite reduced primary group isomorphic to no proper subgroup of itself. Indeed we have

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