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ON DUAL INTEGRAL EQUATIONS AS CONVOLUTION TRANSFORMS

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1. Introduction. C. Fox [1] introduced the following dual integral equations

$$\int_0^\infty H\left(ux\left|\begin{matrix}\alpha_i, a_i\\ \beta_i, a_i\end{matrix}\right.; n\right)f(u)du = g(x), \qquad 0<x<1, \tag{1}$$

and

$$\int_0^\infty H\left(ux\left|\begin{matrix}\lambda_i, a_i\\ \mu_i, a_i\end{matrix}\right.; n\right)f(u)du = h(x), \qquad x>1, \tag{2}$$

where $g(x)$ and $h(x)$ are given and $f(x)$ is to be determined.

This class involves some important equations in mathematical phisics as special cases.

The H functions of order n used in equation (1) are of the form

$$H\left(x\left|\begin{matrix}\alpha_i, a_i\\ \beta_i, a_i\end{matrix}\right.; n\right) = H\left(x\left|\begin{matrix}\alpha_1, a_1\\ \beta_1, a_1\end{matrix} : \begin{matrix}\alpha_2, a_2\\ \beta_2, a_2\end{matrix} : \cdots : \begin{matrix}\alpha_n, a_n\\ \beta_n, a_n\end{matrix}\right.\right) \tag{3}$$

$$= \frac{1}{2\pi i}\int_C \prod_{i=1}^n \left\{\frac{\Gamma(\alpha_i+sa_i)}{\Gamma(\beta_i-sa_i)}\right\} x^{-s}\, ds \tag{4}$$

where $a_i>0$, α_i, β_i are all real, $i=1,2,\cdots,n$, the contour C along which the integral of (4) is taken is the straight line parallel to the imaginary axis in the complex s-plane and all the poles of the integrand of (4) are simple and lie to the left of the line $\sigma=\sigma_0>-\alpha_i/a_i \quad i=1,2,\cdots,n \quad (s=\sigma+i\tau)$. The integral (4), taken along the line, converges if $2\sigma_0\sum_{i=1}^n a_i<\sum_{i=1}^n(\beta_i-\alpha_i)$ and converges absolutely if $2\sigma_0\sum_{i=1}^n a_i<\sum_{i=1}^n(\beta_i-\alpha_i)-1$.

Under the assumptions that the H function of (2) satisfies these conditions with α_i replaced by λ_i and β_i replaced by μ_i $(i=1,2,\cdots,n)$ and a common value of σ_0 can be found for both the H functions, he found a formal solution