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RINGS OF U-DOMINANT DIMENSION $\geqq 1$

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Dedicated to Professor Tadao Tannaka on his 60th birthday.

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Recently Tachikawa [8] has proved that, if R is a ring of dominant dimension $\geqq 1$, then domi. dim. $R_R \geqq 2$ if and only if the injective hull $E(R_R)$ of R_R has the double centralizer property. Our purpose here is to generalize this result which is also the origin of the present paper. We are mainly concerned with double centralizers of finitely-faithful injective modules and examine the double centralizer property of such modules. To this end, we introduce in Section 1 U-dominant dimension for modules, where U is a module. The U-dominant dimension is, roughly speaking, a relative dominant dimension with respect to a given module U (for a definition of dominant dimension, see Kato [3, §1] and Tachikawa [8, §1]). It is shown in Theorem 1 that the double centralizer of a finitely-faithful, injective module U_R over a ring R has always U-dominant dimension $\geqq 2$. On the other hand our main Theorem 2 states that a finitely-faithful, injective right R-module U_R has the double centralizer property if and only if U-domi. dim. $R_R \geqq 2$. The final Section 3 is devoted to the situation that U-domi. dim. $R_R = 1$. Let U-domi. dim. $R_R = 1$, where U_R is finitely-faithful and injective, and let Q be the double centralizer of U_R. Then $R \neq Q$ by Theorem 2. Now let $R \subset Q' \subset Q$ be an intermediate ring between R and Q. Then Theorem 3 states that U-domi. dim. $Q'_{Q'} = 1$ if and only if $Q' \neq Q$. These theorems yield interesting corollaries which generalize results of Mochizuki [5, Theorem 3.1], Tachikawa [7, Theorem 1.4] and Tachikawa [8, Theorem 1.4].

Throughout this paper, rings will have a unit element and modules will be unital. A_R will denote, as usual, the fact that A is a right module over a ring R. If A_R is a module over a ring R, $E(A_R)$ will denote the injective hull of A_R. We adopt the notation that homomorphisms of modules will be written on the side opposite to the scalars.

1. Introduction. Let R be a ring, and U_R a right R-module. A right R-module X_R is called U-torsionless in case $X_R \hookrightarrow \prod U_R$, where $\prod U_R$ is a