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HYPERSURFACES SATISFYING A CERTAIN CONDITION ON THE RICCI TENSOR

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1. Introduction. The Riemannian curvature tensor R of a locally symmetric Riemannian manifold (M,g) satisfies

$$(*) \qquad R(X, Y)\cdot R = 0 \quad \text{for any tangent vectors } X \text{ and } Y,$$

where the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M. A result of K. Nomizu [2] tells us that the converse is affirmative in the case where M is a certain hypersurface in a Euclidean space. That is:

Let M be an m-dimensional, connected and complete Riemannian manifold which is isometrically immersed in a Euclidean space E^{m+1} so that the type number $k(x) \geqq 3$ at least at one point x. If M satisfies condition $(*)$, then it is of the form $M=S^k \times E^{m-k}$, where S^k is a hypersphere in a Euclidean subspace E^{k+1} of E^{m+1} and E^{m-k} is a Euclidean subspace orthogonal to E^{k+1}.

Let R_1 be the Ricci tensor of (M,g). Then condition $(*)$ implies in particular

$$(**) \qquad R(X, Y)\cdot R_1 = 0 \quad \text{for any tangent vectors } X \text{ and } Y.$$

First we have

THEOREM A. *Let M be an m-dimensional, connected and complete Riemannian manifold which is isometrically immersed in a Euclidean space E^{m+1} so that the type number $k(x) \geqq 3$ at least at one point x. If M satisfies condition $(**)$ and has the positive scalar curvature, then it is of the form $M = S^k \times E^{m-k}$.*