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COMPARISON BETWEEN $T(r, f)$ AND $\log M(r, f)$*)

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1. **Introduction.** Let $f(z)$ be a transcendental entire function and let

$$M(r) = M(r, f) = \max_{|z|=r} |f(z)|$$

be the maximum modulus of $f(z)$ on $|z| = r$ and

$$T(r) = T(r, f) = (1/2\pi) \int_0^{2\pi} \log^+ |f(re^{i\theta})| \, d\theta$$

the characteristic function of $f(z)$, where $\log^+|x| = \max(\log|x|, 0)$.

We define the order ρ and lower order λ of $f(z)$ as follows;

$$\rho = \limsup_{r\to\infty} \frac{\log \log M(r, f)}{\log r}, \quad \lambda = \liminf_{r\to\infty} \frac{\log \log M(r, f)}{\log r}.$$

Paley [6] proved that for each $\rho\,(0 \leqq \rho \leqq \infty)$, there is an entire function of order ρ for which

$$\limsup_{r\to\infty} \frac{\log M(r, f)}{T(r, f)} = \infty.$$

On the other hand, it is conjectured that

$$C_f = \liminf_{r\to\infty} \frac{\log M(r, f)}{T(r, f)} \leqq \pi\rho$$

for $1/2 < \rho < \infty$ (see [4, 6]), and it is known that

$$C_f \leqq \pi\rho/\sin \pi\rho$$

for $0 \leqq \rho \leqq 1/2$, and this is the best possible estimate (see [9, 11]).

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