Tôhoku Math. Journ.
21(1969), 249-270.

ON THE ALGEBRA OF MEASURABLE OPERATORS FOR A GENERAL AW^*-ALGEBRA

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(Received November 19, 1968)

1. Introduction. It is an interesting problem in the non-commutative integration theory to construct a "measurable operator" without using unbounded linear operators. From this point of view, we shall extend Berberian's result on "The regular ring of a finite AW^*-algebra" to general AW^*-algebras. S. K. Berberian defined a "closed operator" for a finite AW^*-algebra in algebraic fashion and studied the structure of the "closed operators" [1].

The plan of this paper is as follows. Section 3 is devoted to formulate the notions of "strongly dense domains" and "measurable operators" with respect to a given AW^*-algebra M. Our definitions are closely related to that of [1]. Along the same lines with [1], we shall construct the algebra $\mathcal{C}$ of "measurable operators" for the general AW^*-algebras and study some preliminary algebraic properties of $\mathcal{C}$. Section 5 deals with the spectral theorem for "self-adjoint measurable operators" using the Cayley transform. Theorem 5.1 gives the necessary and sufficient condition for a unitary element in M to be the Cayley transform of some "self-adjoint element" of $\mathcal{C}$. In particular, Lemma 4.1 and Theorem 5.1 play essential rôles in our discussions. In section 6, Theorem 6.2 gives an alternative proof of ([5] Theorem): If $\mathcal{C}$ is regular ([10], Definition 2.2), then M is finite. Theorem 6.3 concerns with the polar decomposition of a "measurable operator" which is one of the main theorems in this paper. Moreover, we shall show that $\mathcal{C}$ is a Baer*-ring in the sense of [6].

Before going into discussions, the author wishes to express his gratitude to Prof. M. Takesaki for calling his attention to the reference [1], and he is also grateful to Prof. J. Tomiyama for useful conversations with him.

2. Notations and Definitions. An AW^*-algebra M is a C^*-algebra satisfying the following two conditions:

(a) In the set of projections any collection of orthogonal projections has a least upper bound.

(b) Any maximal commutative self-adjoint subalgebra is generated by its projections.