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THREE REMARKS ON FUNDAMENTAL GROUPS OF SOME RIEMANNIAN MANIFOLDS

TAKASHI SAKAI

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1. Statement of Results. It is an important object of global Riemannian geometry to study the relation between topological structure and metrical structure of Riemannian manifolds. In this note we shall give some elementary results about the fundamental groups of Sasakian manifolds, Kählerian manifolds, and manifolds of non-positive sectional curvature.

(a) Let M be a regular contact manifold with Sasakian structure (ϕ, ξ, η, g), that is, a regular contact manifold with the metric which is an odd dimensional analgoue of Kählerian metric. In particular, if M is compact, it is a principal circle bundle over a compact Kählerian manifold $'M$, where the contact form η is a connection form of this principal bundle. (S. Sasaki [6]).

Let D denote the distribution defined by $\eta = 0$. Then for a unit tangent vector $X \in D$ we define ϕ-holomorphic sectional curvature $H(X)$ of X by $H(X) = K(X, \phi X)$, where $K(X, \phi X)$ is the sectional curvature of the plane spanned by X and ϕX. It is known that the sectional curvature $K(X, \xi) = 1$ for $X \in D$.

Let $p \colon M \longrightarrow {'M}$ denote the fibering of M stated above, where $'M$ is a compact Kählerian manifold. If we put $'X = p_*(X)$, then $'X$ is the unit tangent vector of $'M$ and X is the horizontal lift of $'X$ with respect to the connection η. We have

$$'H('X) = H(X) + 3 \tag{1}$$

where $'H('X)$ denotes the holomorphic sectional curvature of $'M$. (S. Tanno [7]). Now we get

THEOREM A. *Let M^d be a compact regular Sasakian manifold of dimension d with $h \leqq H(X) \leqq H$ for any unit tangent vector $X \in D$ where h, H are constants. If $h > -3$ holds then the fundamental group $\pi_1(M)$ is cyclic and if $(h+3)/(H+3) > (d-3)/2(d-1)$ holds, then $\pi_1(M)$ is finite cyclic.*

This improves a result of M. Harada ([4]) which has been proved by