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SOME REMARKS ON MINIMAL SUBMANIFOLDS

KATSUEI KENMOTSU

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This note consists of three topics for minimal submanifolds. M denotes an n-dimensional manifold which is minimally immersed in an $(n+p)$-dimensional Riemannian manifold $\bar{M}^{n+p}[c]$ of constant curvature c. In the section 1 we study a linear connection $\widehat{\nabla}$ on the normal vector bundle $N(M)$ which is naturally induced from the connection of the ambient space $\bar{M}^{n+p}[c]$. Let $\widehat{R}$ be the curvature tensor of $\widehat{\nabla}$ and let σ be the square of the length of the second fundamental form of this immersion. Then it is proved that if M is compact, orientable and $\widehat{R}=0$, then

$$\int_M \sigma(\sigma - nc)dv \geqq 0,$$

where dv denotes the volume element of M. It follows that if $\sigma \leqq nc$ everywhere on M, then either

$$(1) \qquad \sigma = 0 \quad (\text{i. e., } M \text{ is totally geodesic}),$$

or

$$(2) \qquad \sigma = nc.$$

The purpose of the section 1 is to determine all minimal submanifolds in a unit sphere $S^{n+p}[1]$ satisfying $\sigma = n$ and $\widehat{R}=0$. The result can be found in Theorem 3.

In the section 2, we study a minimal hypersurface M in $S^{n+1}[1]$. R and R_1 denotes the curvature tensor and Ricci tensor of M, respectively. We will prove that if the Ricci tensor R_1 of M satisfies the condition $R(X,Y)\cdot R_1 = 0$, then, within rotations of $S^{n+1}[1]$, M is an open submanifold of one of the Clifford minimal hypersurfaces:

$$M_{k,n-k} = S^k\left(\sqrt{\frac{k}{n}}\right) \times S^{n-k}\left(\sqrt{\frac{n-k}{n}}\right), \text{ for } k = 0, 1, \cdots, \left[\frac{n}{2}\right].$$