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SOME HYPERSURFACES OF A SPHERE

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1. Introduction. K.Nomizu [2] studied the effect of the condition

(*) $R(X, Y)\cdot R = 0$ for any tangent vectors X and Y

for hypersurfaces M^m of the Euclidean space E^{m+1}, where R denotes the Riemannian curvature tensor and $R(X,Y)$ operates on the tensor algebra at each point as a derivation. P.J.Ryan [4] treated the same condition for hypersurfaces of spaces of non-zero constant curvature. On the other hand, one of the authors [6] discussed the effect of the condition

(**) $R(X, Y)\cdot R_1 = 0$ for any tangent vectors X and Y

for hypersurfaces of the Euclidean space, where R_1 denotes the Ricci curvature tensor.

The condition (*) implies the condition (**).

Recently, P.J.Ryan informed one of the authors that the conditions (*) and (**) are equivalent if the ambient space is of non-zero constant curvature.

In this note we prove

THEOREM. *Let M^m, $m\geqq 4$, be an m-dimensional connected and complete Riemannian manifold which is isometrically immersed in a sphere $S^{m+1}(\bar{c})$ of curvature $\bar{c}$. Then M^m satisfies the condition (**), if and only if M^m is one of the following spaces:*

(i) $M^m = S^m(\bar{c})$; *great sphere*,

(ii) $M^m = S^m(c)$; *small sphere, where* $c > \bar{c}$,

(iii) $M^m = S^p(c_1) \times S^{m-p}(c_2)$, *where* p, $m-p\geqq 2$ *and* $c_1>\bar{c}$, $c_2>\bar{c}$ *such that* $c_1^{-1} + c_2^{-1} = \bar{c}^{-1}$,